SERVICE LIMIT STATES FOR RAILWAY BRIDGES IN NEW DESIGN CODES IAPF AND EUROCODES



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ABSTRACT

The new and enhanced performance needs of bridges for high-speed railway lines have prompted new requirements for design of structures. These have been studied at national and international level within Europe (ERRI, UIC, Eurocode project teams) and have originated new engineering codes for actions and design requirements. Between these we may cite the Eurocodes EN1991-2 (2003) and EN1990-A1 (2005) and the new Spanish code IAPF (2007). An important feature in these codes is the consideration of service limit states. These limit states are unique to railway bridges and are often the critical features conditioning the design. Among these limits are the maximum of displacements and stresses in the rail related to trackbridge interaction, and the limit of accelerations at the track. It must be stressed that some of these service limit states are indeed ultimate limit states related to safety of rail traffic, and hence of the utmost importance. In this work we shall review these limitations, the methods proposed for calculation, and their relevance for design.

1. INTRODUCTION: NEW DESIGN REQUIREMENTS FOR HIGH SPEED RAILWAY BRIDGES

New high speed railway lines are developing at a fast pace in some European countries. In particular, in Spain the plan for transport infrastructure (PEIT 2005-2020) devotes 78000 Million \notin to high speed railways out of a total investment of 241000 Million \notin .

Railway bridges for the new high speed lines introduce a number of design requirements which cause significant differences not only with road bridges but also with other railway bridges in conventional freight or passenger lines. A first and obvious requirement arises directly from the higher speed of traffic actions. These not only produce a higher individual effect (measured through the *impact factor* Φ), but more importantly for speeds above 200 km/h the risk of *resonance* appears. As a result dynamic analyses must be carried out in general, and furthermore these considerations must be taken into account in the design of structural characteristics. In particular, some structural types such as short span isostatic bridges have been shown to originate high levels of vibration exceeding the limits for comfort and safety.

Furthermore the stricter requirements for the high speed lines (e.g. maximum gradients, minimum radii) and geometrical quality originate line layouts in which more and longer viaducts are necessary. This is particularly important in regions with rugged terrain like the Iberian Peninsula. For instance, the new lines in Spain include a number of viaducts longer than 1000 m, some even reaching 3000 m.

The consideration of interaction between bridge and track introduces additional requirements to be met by railway bridges. These stem from limitations of stresses in the rail as well as maximum values of relative displacements in deck joints. The magnitude of the track-bridge interaction effects increases with the continuous length (expansion length L_T) of the deck. As a result, in short bridges these requirements are not especially restrictive. However in long viaducts, of the lengths commonly required by high speed line layouts, they prove to be an important restriction and must be considered at early stages of design. One option which has been adopted in some lines is to limit the continuous length of the decks, splitting long viaducts into individual simply supported structures. This option may be overly restrictive. Moreover, it bears the disadvantage expressed above that simply supported beam structures develop generally higher vibration response to traffic loads. Furthermore, the optimization of construction procedures in some cases makes advantageous the progressive launching or pushing of a long continuous deck.

The above considerations have led the administration and the infrastructure manager (ADIF) in Spain to allow the construction of long continuous viaducts, exceeding in some cases 1000 m length. The bases for calculation and associated limits are defined in IAPF (2007) and at a European level in EN1991-2 (2003).

In the remaining of this paper, in section 2 we shall firstly review some design considerations for high speed railway bridges, with special emphasis on those originating from dynamic behaviour. Following, in section 3 we shall review the methods and requirements for track-bridge interaction in the codes, focusing on the new IAPF (2007) compared to EN1991-2 (2003). Here the serviceability limit state checks regarding deformations of the deck will be discussed critically. The paper finishes with a summary of the main conclusions in section 4.

2. DESIGN CONSIDERATIONS FOR RAILWAY BRIDGES FROM DYNAMIC EFFECTS

Dynamic response of railway bridges is a major factor for design and maintenance, especially in new high speed railway lines. The main concern is the risk of resonance from periodic action of moving train loads. In cases when such risk is relevant (e.g. for speeds above 200 km/h) a dynamic analysis is mandatory.

The new engineering codes [EN1991-2 (2003), EN1990-A1 (2005), IAPF (2007), FS. (1997)] take into account these issues and define the conditions under which dynamic analysis must be performed. They provide guidelines for models, types of trains to be considered, and design criteria or limits of acceptance [Goicolea, JM. (2004)].

Resonance for a train of periodically spaced loads may occur when these are applied sequentially to the fundamental mode of vibration of a bridge and they all occur with the same phase, thus accumulating the vibration energy from the action of each axle. If the train speed is v, the spacing of the loads D and the fundamental frequency f_0 , defining the *excitation wavelength* as $\lambda = v/f_0$, the condition for critical resonant speeds may be expressed as [EN1991-2 (2003)]:

$$\lambda = \frac{D}{i}, \quad i = 1, \dots 4. \tag{1}$$

Although the basic dynamic phenomena due to moving loads are known since long (e.g. see the book by Fryba, L (1972)) it has not been until recently that resonance phenomena in bridges from high speed traffic have been well understood and practical methods of analysis developed [ERRI D214. (2002), Domínguez J. (2001)]. From a technical point of view a number of methods for dynamic analysis are available for engineering practice. Briefly, the available methods are a) dynamic analysis with time integration and moving loads; b) dynamic analysis with time integration and bridge-vehicle interaction; and c) dynamic envelopes based on train signature methods. Rather than discussing these methods here, for which a complete description is given elsewhere [Domínquez J. (2001), Goicolea, JM (2004)] we shall focus on the relevance of dynamic effects for structural designs.

In railway bridge design often the most restrictive conditions in practice are the Serviceability Limit States (SLS) [EN1990-A1 (2005), Nasarre J (2004)] (maximum acceleration, rotations and deflections, etc.). Accelerations must be independently obtained in the dynamic analysis. In the example shown in Figure 1, for a short span simply supported bridge, both maximum displacements and accelerations are obtained independently and checked against their nominal (LM71) or limit values respectively. It is clearly seen that for a resonant train speed the deflection limits are above the LM71 nominal values, resulting in an impact factor Φ greater than unity. A more severe effect is the accelerations which surpass by far the limits, thus invalidating this (purely theoretical) design. Further details may be seen in Goicolea, JM. (2004). These results have been obtained both with moving loads and with bridge-vehicle interaction, showing that the gain of considering this latter and more advanced model, albeit significant in this case, still yields a non acceptable value.



Figure 1: Calculations for simply supported bridge from ERRI D214. (2002) catalogue $(L=15 \text{ m}, f_0=5 \text{ Hz}, \rho=15000 \text{ kg/m}, \delta_{\text{LM71}}=11 \text{ mm})$, with TALGO AV2 train, for non-resonant (360 km/h, top) and resonant (236.5 km/h, bottom) speeds, considering dynamic analysis with moving loads and with train-bridge interaction. Note that the response at the higher speed (360 km/h) is considerably smaller than for the critical speed of 236.5 km/h. The graphs at left show displacements, comparing with the quasi-static response of the real train and the LM71 model, and those at right accelerations, compared with the limit of 3.5 m/s² [EN1990-A1 (2005)]

In order to consider the resonant velocities for dynamic calculations, these must be performed generally for all possible speeds. The results may be presented as envelopes of resulting magnitudes in these velocity sweeps. Following we present a typical set of such calculations showing the fact that generally resonance may be much larger for short span bridges. In this representative example, Figure 2 shows the normalised displacement response envelopes obtained for ICE2 train in a velocity sweep between 120 and 420 km/h at intervals of 5 km/h. Calculations are performed for three different bridges, from short to moderate lengths (20 m, 30 m and 40 m). The maximum response obtained for the short length bridge is many times larger that the other. The physical reason is that for bridges longer than coach length at any given time several axles or bogies will be on the bridge with different phases, thus cancelling effects and impeding a clear resonance. We also remark that for lower speeds in all three cases the response is approximately 2.5 times lower than that of the much heavier nominal train LM71. Resonance increases this response by a factor of 5, thus surpassing by a factor of 2 the LM71 response.

A significant reduction of vibration is obtained in short span bridges under resonance by using interaction models. This may be explained considering that part of the energy from the vibration is be transmitted from the bridge to the vehicles. However, only a modest reduction is obtained for non-resonant speeds. Further, in longer spans or in continuous deck bridges the

advantage gained by employing interaction models will generally be very small. This is exemplified in Figure 2, showing results of sweeps of dynamic calculations for the three said bridges of different spans. As a consequence it is not generally considered necessary to perform dynamic analysis with interaction for project or design purposes.



Figure 2: Normalised envelope of dynamic effects (displacement) for ICE2 high-speed train between 120 and 420 km/h on simply supported bridges of different spans (*L*=20 m, f_0 =4 Hz, ρ =20000 kg/m, δ_{LM71} =11.79 mm, *L*=30 m, f_0 =3 Hz, ρ =25000 kg/m, δ_{LM71} =15.07 mm and *L*=40 m, f_0 =3 Hz, ρ =30000 kg/m, δ_{LM71} =11.81 mm). Dashed lines represent analysis with moving loads, solid lines with symbols models with interaction. Damping is ζ =2% in all cases

The above results are not merely theoretical considerations. It has been seen in practice that they reflect accurately the vibrations taking effect in real high speed railway bridges. To show this we comment some experimental results on an existing high speed bridge. Figure 3 shows the measured resonant response in the bridge over the river Tajo in the Madrid-Sevilla HS line. The bridge consists of a sequence of simply supported isostatic decks with spans of 38 m. The dynamic amplification in this case is noticeable. In spite of this, design responses keep within required limits. However, it is clear that the dynamic performance could be improved by a different structural design.



Figure 3: Time history of displacements at centre of span in viaduct over Tajo river in HS line Madrid-Sevilla. Simply supported deck with span 38 m, damping ratio $\zeta = 1.65\%$. Left graph shows measurements [MFOM (1996)], right graph analytical calculations [Domínguez J. (2001)]. Horizontal scale is time (s), vertical scale displacements (mm)

Another well-known issue is the fact that dynamic effects in indeterminate structures, especially continuous deck beams, are generally much lower than isostatic structures [Domínguez J. (2001)]. The vibration of simply supported beams is dominated clearly by the first mode, and moreover only the loads on the span under consideration excite the motion at a given instant. This makes much more likely a resonant phenomenon, whenever condition (1) is met. On the contrary, the vibration of continuous beams includes significant contributions of a number of modes, and loads on other spans excite the motion of the span under consideration. As a result, the algebraic sum of the effects tends to cancel to a large extent.

We show a practical example of this effect in a student project for the Arroyo del Salado viaduct [Sanz, B (2005)] on the Córdoba-Málaga High speed line, with a total length of 900 m and 30 spans of 30 m each. The section is a prestressed concrete box deck, and the proposed solution was a continuous beam deck cast in-situ. The comparison of this solution with a corresponding simply supported multiple span viaduct is shown in Figure 4, where it may be seen the much better performance in terms of dynamic response of the continuous beam deck.



Figure 4: Summary of dynamic analysis envelopes with universal HS trains HSLM showing maximum accelerations in the deck. The graph on the left corresponds to the proposed design as continuous beam, which satisfies the requirement for accelerations $a_{max} < 3.5 \text{ m/s}^2$. The right graph corresponds to a simple supported bridge with the same deck section; in this case the requirement for maximum accelerations is not fulfilled for high speeds.

Finally, we discuss the consideration of different high speed train types. The existing trains in Europe are defined in EN1991-2 (2003), IAPF (2007), and may be classified into conventional (ICE, ETR-Y, VIRGIN), articulated (THALYS, AVE, EUROSTAR) and regular (TALGO). Variations of these trains which satisfy interoperability criteria have been shown to covered by the dynamic effects of the High Speed Load Model (HSLM), a set of universal fictitious trains proposed by ERRI D214. (2002). The use of this new load model is highly recommended for all new railway lines, and incorporated into codes EN1991-2 (2003) and IAPF (2007). More importantly, consideration of HSLM model is mandatory for interoperable lines following the European Technical Specifications for Interoperability (TSI) in high speed lines [EC (2002)].

A useful way to compare the action of different trains and to evaluate the performance of HSLM as an envelope is to employ the so-called *dynamic train signature* models. These develop the response as a combination of harmonic series, and establish an upper bound of this sum, avoiding a direct dynamic analysis by time integration. Their basic description may be

found in [ERRI D214. (2002)]. They furnish an analytical evaluation of an upper bound for the dynamic response of a given bridge. The result is expressed as a function of the *dynamic signature* of the train $G(\lambda)$. This function depends only on the distribution of the train axle loads. Each train has its own dynamic signature, which is independent of the characteristics of the bridge. The above expressions have been applied in Figure 5 to represent the envelope of all real existing HS trains in Europe, together with the envelope of HSLM.



Figure 5: Envelope of dynamic signatures for European HS trains, together with the envelope of signatures for High Speed Load Model HSLM-A, showing the adequacy of this load model for dynamic analysis

3. TRACK-BRIDGE INTERACTION IN CODES IAPF (2007), EN1991-2 (2003)

3.1 Nature of phenomenon and effects to be evaluated

Track-bridge interaction originates from the fact that longitudinal forces in long welded rail are transmitted both by the structure and the rail to the fixed points at piers or abutments. Furthermore, at joints in the deck there may be structural deformations which could modify the geometry of the track and thus endanger the safety of traffic.

For short bridges this issue is not critical and in fact given certain conditions the calculation of the nonlinear models described below may be avoided. However, as has been said above, in high speed lines bridges and viaducts of substantial length are common and hence the issue of track-bridge interaction becomes a critical issue.

The basic interpretation and methods agreed internationally are contained in the leaflet by UIC (1999), which summarises the results by ERRI subcommittee D213. Both the Spanish code IAPF (2007) and the Eurocode EN1991-2 (2003) follow generally the recommendations of the said UIC leaflet. They both contain a section describing specifically the objectives of this evaluation, the actions and models to consider and the design requirements. In what follows we describe in summary the main principles, which are common between both codes and, wherever appropriate, underline and comment specifically the differences or additions.

In both codes it is stated that consideration of track-bridge interaction is necessary in order to evaluate the following effects:

- Forces transmitted to piers and abutments from combined actions of structure and track;
- Rail stresses due to variable actions, in particular thermal actions, braking and acceleration longitudinal forces and vertical traffic loads;
- Relative movements and deformations at the ends of the deck due to the above variable actions.

3.2 Models to employ in calculation

Several types of structures may be considered from the point of view of track-bridge interaction: a) single deck bridges, be this with one isostatic span or with a multiple span continuous beam, with a fixed bearing at one end; b) continuous beams with multiple spans with a fixed bearing at an intermediate point of the bridge; and c) multiple isostatic spans with fixed bearings ate the end of each span.

The general type of model to be considered is depicted schematically in Figure 6, for case a) above. This model considers the track and the deck (both considered deformable elastically), the piers, and the abutments which may also be flexible. A key aspect in the model is the proper consideration of the interaction forces between rail and deck, in the figure represented through *generalised springs*, which as we shall see below are of nonlinear nature. Finally, in the figure a rail expansion device which would signify a longitudinal discontinuity for the rail is also shown.



Figure 6: Model to be considered for track-bridge interaction, in a simple case with one deck. The figure shows a deck with one fixed point and two sliding supports, nonlinear "generalised springs" which model the longitudinal interaction between track and deck, and an optional rail expansion device at one end (figure translated from IAPF (2007))

A characteristic value of these models is the so-called *expansion length* L_T . In the example shown it would be simply the length of the deck between the fixed support on one abutment and the free-sliding joint on the other abutment. The greater the value of L_T the greater interaction effects will be introduced at the free sliding joint.

When expansion lengths are large the rail stresses may be reduced by the introduction of rail expansion devices. In such case, the horizontal deck forces would be transferred integrally to the fixed bearing, alleviating the effects on the rail. However, rail expansion devices are generally undesirable from the point of view of track engineering and maintenance. Expansion lengths of the order of 100 m may generally be accommodated without resorting to rail

expansion devices. Expansion lengths of the order of 300 m to 400 m will very probably necessitate at least one rail expansion device. Expansion lengths greater that this may necessitate at least two expansion devices, say with a fixed point at the center, or other structural solutions.

The above mentioned nonlinear generalised springs are defined with bilinear laws, of the type shown in Figure 7. The first branch represents an elastic behaviour, whereas after a given displacement u_0 the sliding limit is attained and the constant resistance k is developed. The Eurocode EN1991-2 (2003) leaves the values of (u_0, k) open, to be defined in national annex or other project specifications. The code IAPF (2007) defines values for u_0 between 0.5 and 2.0 mm, and for k between 20 and 60 kN/m, depending on the type of track, vertical load etc.



Figure 7: Force-displacement interaction law between track and deck. The parameter u_0 defines the maximum relative displacement at which sliding starts, with a plasticity or friction-type resistance defined as *k*. Particular values of u_0 and *k* are defined in IAPF (2007) for different types of tracks and situations. (figure translated from IAPF (2007))

The structural model described may be developed within a discretised computer model of the structure with nonlinear material capabilities, such as finite element or other numerical programs. This program must have the capability to solve the resulting set of nonlinear algebraic equations, generally using an iterative procedure by Newton-type iterations until convergence is reached. An essential characteristic of nonlinear models is that superposition of actions is not valid; hence for each calculation the complete set of actions must be applied in the correct sequence to the model. In particular for this case, for each scenario selected the thermal actions would be applied first and then the vertical and longitudinal traffic actions.

For short expansion lengths L_T both codes allow simplified procedures for calculation. In particular, for $L_T \leq 40$ m in EN1991-2 (2003) or $L_T \leq 60$ m (steel) – 90 m (concrete) in IAPF (2007) it may be considered that rail expansion devices are not needed, without a full justification by the nonlinear models above described. For somewhat longer expansion lengths, $L_T \leq 110$ m, the code IAPF (2007) refers to the simplified procedures defined in UIC (1999) based on charts for evaluating the interaction.

Furthermore, the Eurocode EN1991-2 (2003) allows the simplification, for evaluating forces in rails and bearings, of combining linearly the effects of the different actions. As has been said before, strictly speaking this linear combination is not valid; however for computation of forces in general a conservative result will be obtained. This is not generally the case for

calculation of deformations, which may be underestimated using this simplification. In the Spanish code IAPF (2007) this simplification is not considered.

3.3 Design criteria

The maximum additional stresses in the rails from the variable actions (thermal and traffic loads) are limited to 72 MPa (compression) or 92 MPa (tension). It is understood that these stresses would apply on top of the existing stresses in the long welded rail, which amount to approx. 105 MPa for a maximum temperature increment $\Delta T = 50^{\circ}$ C.

Regarding the deformation of the deck, it is required to limit the relative movements at the end of the deck in sliding joints (e.g. between end of the deck and abutment). The following requirements are defined, all related to the said relative movements:

- The horizontal movement from braking and acceleration forces must be $\delta_B \le 5$ mm. (called δ_2 in IAPF (2007)). Figure 8 shows a schematic representation of this movement.



- Figure 8: Maximum longitudinal relative displacement from braking or acceleration actions (δ_2 in IAPF (2007) or δ_B in EN1991-2 (2003)) between two ends at a joint (figure from IAPF (2007))
- The horizontal movement from vertical traffic loads must be $\delta_H \leq 5$ mm. This movement originates mainly from bending, which produces horizontal movement at points eccentric from the neutral axis.

In IAPF (2007) this movement, which is called δ_3 , is more precisely defined to be computed not only from the bending caused by vertical traffic loads but also from eccentric horizontal longitudinal loads (i.e. braking or acceleration acting on the rail surface), which also introduce bending moments in the deck. Figure 9 shows a schematic representation of this movement.



Figure 9: Maximum horizontal relative displacement from bending due to vertical or eccentric horizontal actions (δ_3 in IAPF (2007) or δ_H in EN1991-2 (2003)) between two ends at a joint (figure from IAPF (2007))

- The vertical movement from bending and other effects must be $\delta_V \leq 2$ mm. This limitation holds for lines with train speeds above 160 km/h which is the case for high speed.

In the Spanish code IAPF (2007) again this limit (called δ_4) is more precisely defined not as vertical but as normal to the rail within a vertical plane. Figure 10 shows a schematic representation of this movement. As may be seen there could be a noticeable difference between the normal movement which actually alters the track geometry in a track with gradient, and the vertical movement which would be zero in this case. For a long viaduct this difference may be critical.



Figure 10: Maximum relative normal displacement in vertical plane from variable actions (δ_4 in IAPF (2007) or δ_V in EN1991-2 (2003)) between two ends at a joint (figure from IAPF (2007)).

Furthermore to the above requirements, the following limit is defined in IAPF (2007), but not in the Eurocode EN1991-2 (2003):

- The relative movement between rail and deck (or between rail and abutment platform) must be $\delta_1 \leq 4$ mm under the actions for acceleration and braking. This requirement may also be found in UIC (1999).

The above design requirements both for stresses in the rail as well as for movements at the ends of the deck represent serviceability limit states (SLS) for the structure. However, in this case the importance of these limit states is paramount, as they represent ultimate limit states (ULS) for the railway traffic. It must be clearly understood by any structural engineer that these design criteria are often the critical requirements for railway bridges, contrary to the case for road bridges. This has been clearly set out in the paper by Nasarre J (2004).

4. CONCLUSIONS

Considering the above, we summarise the following final remarks:

- In high speed railway lines it is common to be faced with bridges or viaducts of considerable length, for which special consideration needs to be made at early stages of design to track-bridge interaction effects.
- The reduction of dynamic effects is more favourable for continuous beams and for long spans; these factor again favour the consideration of long decks with potential problems for track-bridge interaction.
- The proper consideration of track-bridge interaction requires a nonlinear structural model, which requires careful elaboration and checking on the part of adequately skilled structural engineers. Simplifications to this model must be carefully justified and only employed when clearly conservative.

 Both the Eurocode EN1991-2 (2003) and the new Spanish code IAPF (2007) contain similar sets of recommendations for the models and design requirements. These criteria originate from the report UIC (1999).

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