

NEW DYNAMIC ANALYSIS METHODS FOR RAILWAY BRIDGES IN CODES IAPF AND EUROCODE 1

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Abstract.

Bridges and viaducts for high speed trains are subject to demanding dynamic loads, as to the classical effect of the moving (single) load the dynamic resonance appears for speeds above 220 km/h. The classical methods for evaluation of dynamic impact factors available in engineering, reflected in the codes of practise existing until recently, do not cover this possibility of resonance. The design of such structures requires dynamic calculations which are the object of this paper. In it we cover a general revision of available methods for calculation, as well as a description of the provisions in the new (draft) codes IAPF [13] and Eurocode 1 for actions on bridges [22].

Additionally, some recent research results obtained by our group are presented for high speed traffic loads on bridges. The object of these studies is diverse: sensitivity to integration time-step in modal analysis, simplified torsion analysis, evaluation of the bridge-vehicle interaction in isostatic bridges and a proposal of a simplified method for dynamic analysis of portal frames. All of these topics originate from issues in the application of the new regulations for high speed lines, and are oriented toward being of practical use to designers of railway bridges.

1 INTRODUCTION AND MOTIVATION

The construction of new transport infrastructure has experienced a boom in the last few years in Spain and in other European countries. The main part of the investment in Spain has been dedicated to high speed railway lines. This chapter has also been dominant in other neighbouring countries such as France. These new railway lines are a very competitive alternative for transport between cities at intermediate distances. Currently in Spain there is one line in operation between Madrid and Seville, in early 2003 the inauguration of the section Madrid-Lérida is foreseen within the line Madrid-Barcelona-French border. Several other lines have been decided between Cordoba-Málaga, Madrid-Segovia-Valladolid, the new access Madrid-Valencia-Murcia and Madrid-Toledo, these being in different states of development (award of contracts, project or construction).

This important engineering activity highlights one of the main structural aspects associated specifically to the design of bridges and structures in railway lines: the dynamic effects due to moving loads from train traffic. The relevance of the dynamic response has been known since the early stages of railways, having been considered as one of the design requirements for the structures. This phenomenon propitiated the study of the basic phenomenon of a moving load on a simply supported beam, whose classical solutions were developed (between others) by Timoshenko [14]. More recently, the works of Fryba [15, 16] have gathered very diverse models and features of the dynamics of railway bridges. Finally, one must cite the notable contributions performed in Spain by Alarcón [1, 2].

The design codes existing up to now [18, 21, 19] for design of railway bridges consider the dynamic response through an *impact factor*, which represents the increase in the dynamic response with respect to the static one for a *single moving load*.

According to this coefficient, the dynamic increment [18] reaches a maximum value of $\varphi' = 1.32$, for an ideal straight track (without irregularities). The impact factor will be finally obtained as the envelope

$$\Phi = \max(1 + \varphi' + \varphi''),$$

where this last factor (φ'') stems from the effect of track irregularities.

As a representative example, consider the case of a point load of 195 kN, corresponding to an axle of the engine of the high speed train ICE2, crossing a simply supported bridge with span $L = 15$ m at a constant speed. The remaining mechanical parameters of the bridge are the mass per unit length $\bar{m} = 15$ t/m, flexural stiffness $EI = 7694081$ kN/m², fundamental frequency (first mode of vibration) $f_0 = 5$ Hz and damping rate $\zeta = 2\%$. This bridge belongs to a catalogue of isostatic bridges employed by ERRI in [7] for dynamic calculations. The result of the dynamic analysis at speed $v = 220$ km/h is shown in figure 1, where the maximum dynamic deflection is 2.80 mm. Varying the velocity of the load one may perform a sweep in velocities, for which figure 2 shows the maximum dynamic deflection for each velocity. The absolute maximum within this sweep is $\delta_{\max} = 3.02$ mm for $v = 330$ km/h. Taking into account that the static deflection is $\delta_{\text{est}} = PL^3/(48EI) = 1.78$ mm, this yields a dynamic factor of $\Phi_{\text{real}} = 1.69$. This dynamic factor is covered by the design value prescribed in [18], which for this case results

in $\Phi_{\text{UIC}} = 1 + \varphi' = 2.16$ (not considering the effect of track irregularities).

from the previous result one may conclude that consideration of the impact factor Φ is sufficient for taking into account the dynamic effect of a single moving load. Let us consider now an (ideal) load train, consisting of 10 axles of equal load to that considered above, with a uniform separation between them of $D = 16$ m. The (dynamic) response obtained for two velocities of circulation ($v = 288$ km/h and $v = 360$ km/h) is shown in figure 3. Notice that the response is much higher for the lesser of the above two speeds, indicating that in this case a resonance phenomenon occurs, which does not increase with the train velocity but rather appears at certain critical velocities. In figure 4 the result of the sweep in velocities for this case of the load train is shown, where the maximum corresponds to resonance at a critical velocity of 288 km/h.

The interpretation of this resonant phenomenon is simple: the frequency of application of the cyclic loads due to the axles for $v = 288$ km/h, taking into account their uniform spacing, is $f_P = v/D = 5$ Hz. The coincidence of this excitation frequency with that of the fundamental vibration mode of the bridge ($f_P = f_0 = 5$ Hz) determines the appearance of resonance.

Another (equivalent) manner to interpret resonance is through the so called *wavelength* of excitation,

$$\lambda = \frac{v}{f_0}.$$

Resonance occurs when the characteristic length D_k of separation between axles coincides with a multiple of the said wavelength:

$$\lambda = \frac{D_k}{i}, \quad i = 1, 2, 3, 4. \quad \Rightarrow \quad \text{resonance}$$

In our case, $\lambda = 16$ m, hence the previous condition is clearly fulfilled as it coincides with the regular distance between axles.

Hence it must be clear that the impact factor Φ does not take into account the possible resonance that would occur as a result of the cyclic repetition of loads. However, it must be mentioned that for the bridge frequencies in practise and for the regular distances between axles of real trains resonance has not occurred in practise... until the appearance of high speed trains!

Indeed, for speeds above 200 or 220 km/h, for the regular distances between axles of current railway cars (between 13 and 20 m)—which for a given velocity are the factors which determine the cyclic frequency of the applied loads—resonant phenomena may start to occur. As a real life example, in figure 5 the measured resonant response obtained in the Tajo viaduct is shown, for a speed of 219 km/h [3]. In [4] this case is analysed in greater detail, performing the dynamic calculation with a simple model which nevertheless coincides very well with experimental measurements (figure 6). These resonant measurements would have resulted even more pronounced for a longer (double composition) train, with a greater number of axles.

The Tajo viaduct in the AVE HS line consists of isostatic simply supported spans of $L = 38$ m, whose fundamental frequency of $f_0 = 3.31$ Hz. Hence, for the speed given the excitation wavelength is $\lambda = 18.4$ m, very close to the characteristic regular distance between bogies in the AVE, $D_k = 18.7$ m.

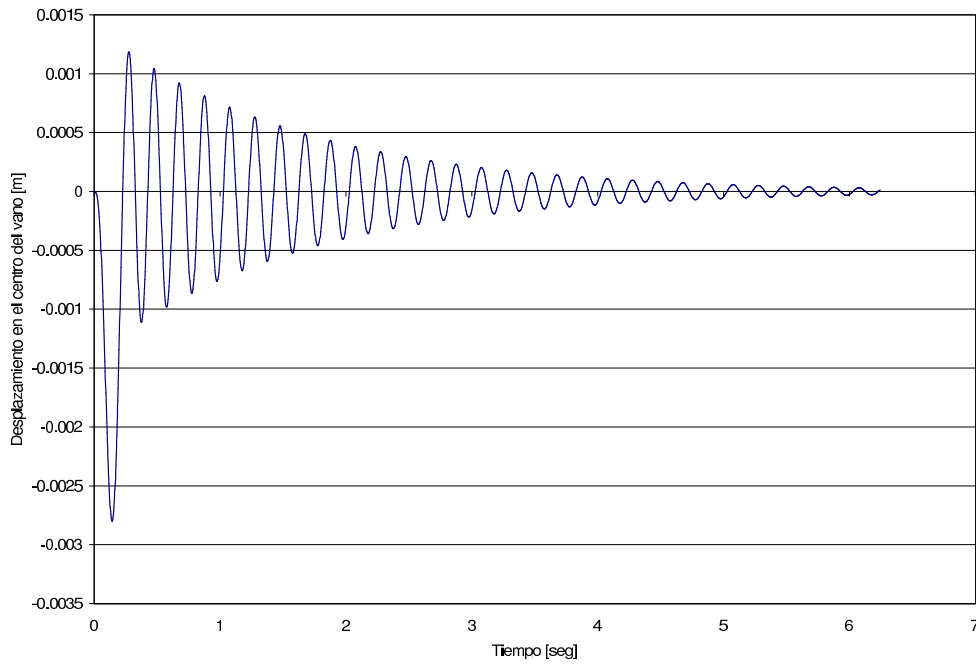


Figure 1. *Dynamic response of a simply supported ERRI bridge $L = 15$ m under a single moving load, $P = 195$ kN, at a speed of 220 km/h.*

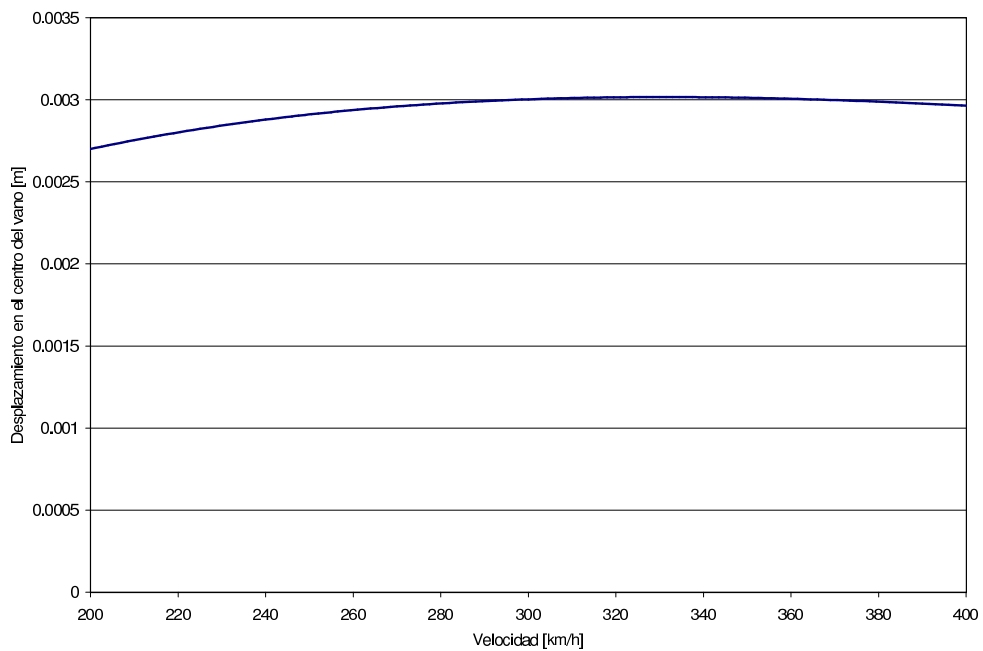


Figure 2. *Maximum deflection at centre of span as a function of load speed. Single moving load, simply supported beam, $L = 15$ m, $\zeta = 2\%$.*

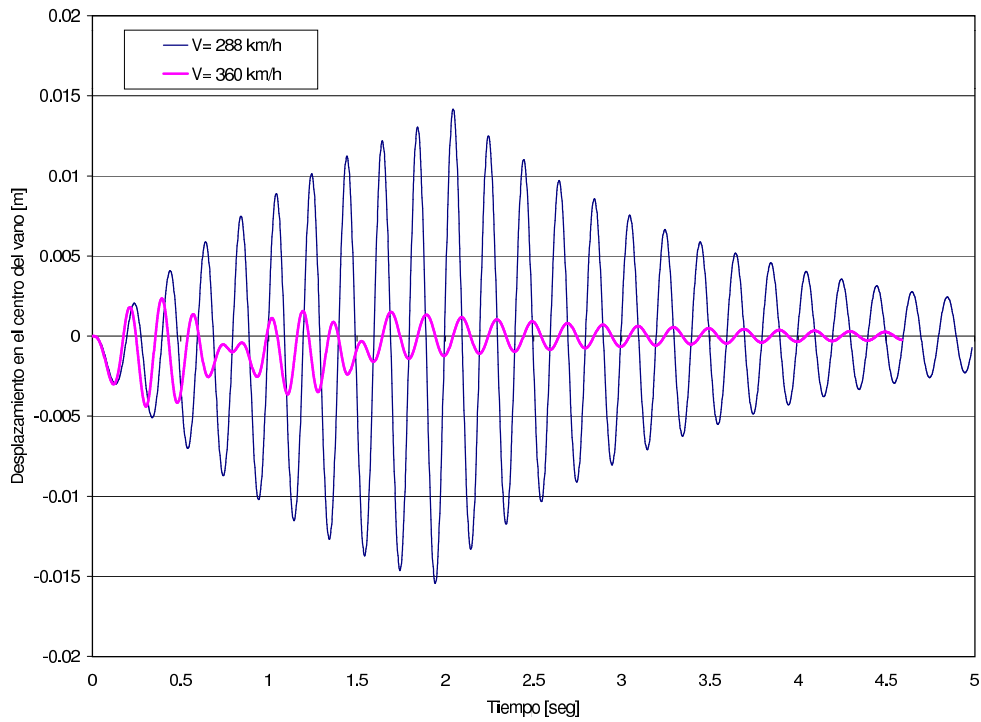


Figure 3. Maximum displacement at centre of span as a function of time for train speeds of $v = 288$ km/h and $v = 360$ km/h.

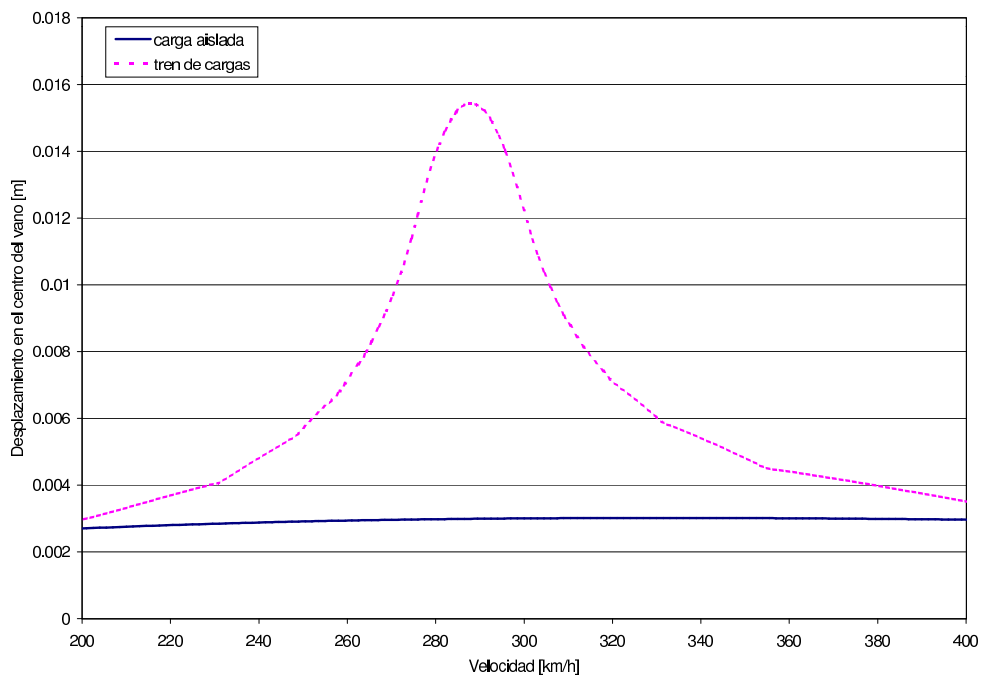


Figure 4. Maximum displacement at centre of span as a function of train speed for the case of single moving load and load train.

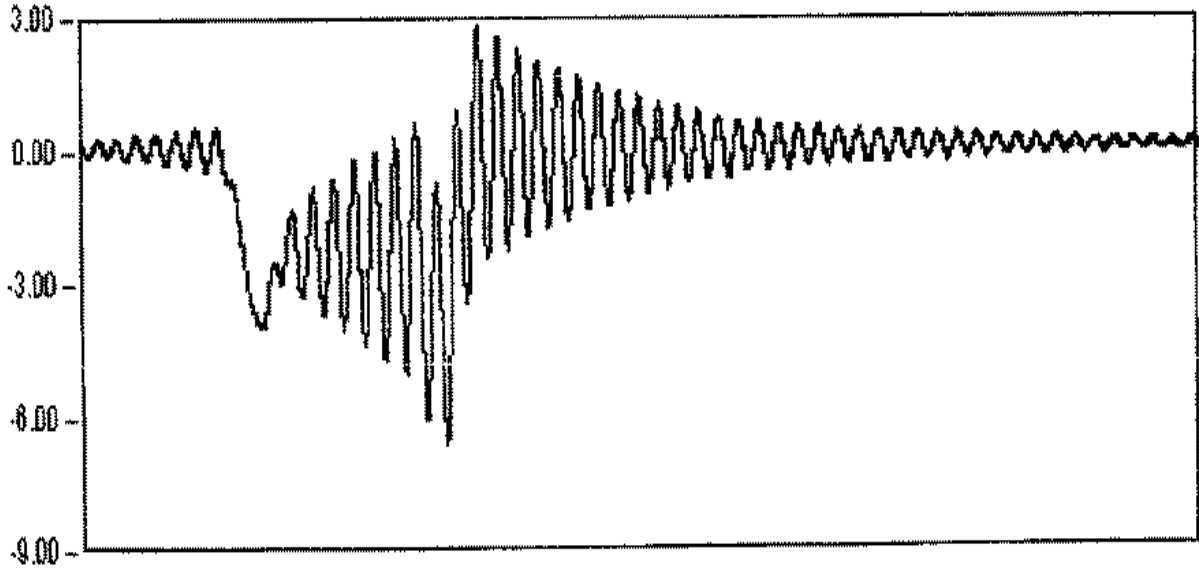


Figure 5. Deflections measured in Tajo viaduct (HS line Madrid-Seville) crossed by AVE train (simple composition) with $v = 219\text{km/h}$ [3].

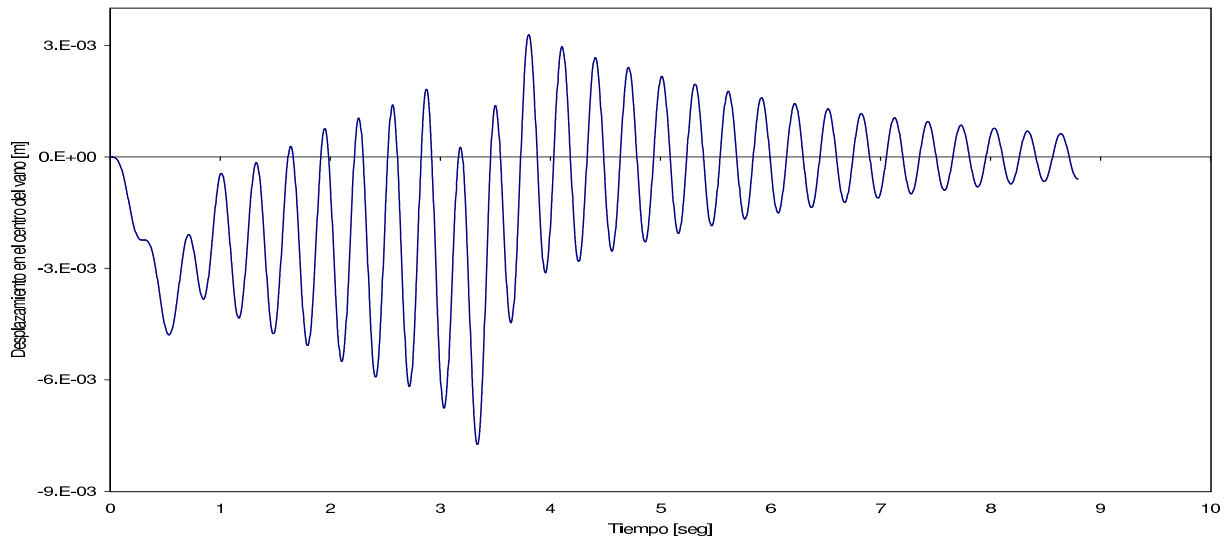


Figure 6. Displacements obtained with a dynamic model with direct time integration and moving loads for the Tajo viaduct, for a speed of AVE HS train of $v = 219\text{ km/h}$ (simple composition).

In summary, resonance in railway bridges, in spite of being a well known feature within dynamic response of systems, has not been included prior to now in engineering design codes. As a consequence, it has not been considered in project, except for the margin provided by the safety factors.

The technical problem posed in structural engineering has far-reaching implications: society demands the realisation of a great number of structures and bridges for the new high speed lines, but analysis methods and design provisions which are reliable, practical and sufficiently validated are not available in the engineering design codes. Some new design codes have recently started to remedy this lack, between these one may point out the Italian code [20], the new draft of Eurocode 1 [22] and the new Spanish draft [13]. Eurocode 1

However, in spite of these new proposals which do consider resonant phenomena, there is still a lack of practical knowledge of the implications of dynamic effects on numerous structural systems in railway bridges, for which reason in our view an important research effort is essential. On the other hand, in spite that from a technical viewpoint a number of calculation models are available, based on linear structural dynamics, it can be said that these methods are not known well enough at this moment by engineers in charge of design and project.

The simplest models with regard to their application are those based on the decomposition as a harmonic series of the dynamic response, and the consequent establishment of upper bounds of these sums through analytic procedures [7, 4]. The drawback associated to these models is that their application is restricted to isostatic structures (e.g. simply supported decks in which dynamic response may be adequately defined by one mode of vibration). They cannot be applied (directly at least) to statically redundant structures. This type of models is described briefly in section 2.4. In the report [12], described in section 7, an analogy is proposed which allows to extend the above models to certain redundant structures such as portal frames in railway underpasses.

The next type of models available for analysis are those based on direct dynamic calculation, with time integration of the response for each degree of freedom, for a set of moving loads representative of the axles of the train [4]. These procedures may be implemented within finite element codes, for which the data preparation (preprocessing) stage for the moving loads may be the the most cumbersome. These models are discussed in section 2.2.1, and with them the dynamic analysis of arbitrary structures may be performed. In certain cases of simple structures (isostatic beams, or continuous beams of two or three vanes) it is possible to apply this procedure also through an analytical *exact* definition of the modes.

Finally the most complete models are those which consider jointly the vibration of the structure as well as that of the vehicle. The latter is taken into account through springs and dampers for the suspensions and the masses and connections provided by the vehicle boxes. Some of these models with *vehicle-structure* interaction are described in section 2.3, having been discussed in greater detail in [4] and references there cited. Obviously, these models are also apt to analyse arbitrary structures, provided the dynamic characteristics of the real trains are known, something which unfortunately is not always the case. This possibility has the drawback of greater calculation costs and an increased complexity in the definition of the model. From the

point of view of research they are interesting or even often indispensable, however it is not reasonable to apply these methods for standard design calculations.

The different engineering design codes prescribe the obligation of specific dynamic analyses when the simplified methodologies are not applicable, a case which is increasingly frequent in structures within high speed lines. Thus, with the view put in obtaining practical and sufficiently validated methods for calculation which facilitate the work of engineers designing railway bridges as well as those responsible for maintenance, several calculation procedures, which complement existing methods, have been proposed recently. These have been employed mostly in the academic research community, the mathematical models involved often exhibit an excessive complexity and calculation cost for project stages. The need for obtaining simplified methods has hence been a goal of engineering research groups, in order to provide calculation tools which are sufficiently precise and contribute to a better comprehension of the dynamic behaviour of structures. The practical studies and applications developed by the authors which are presented in the final sections of this paper follow this line of motivation. These studies focus on the sensitivity to time-step in modal analysis, consideration of torsion, evaluation of vehicle-structure interaction in isostatic bridges, and finally a proposal for a simplified dynamic calculation method for portal frame underpasses.

The consideration of the vehicle-structure interaction models discussed in section 2.3 produces a reduction of the effects due to the existence of mechanisms which permit energy dissipation (dampers) or systems which interchange energy between structure and vehicle (suspension springs). For non resonant situations or statically redundant bridges, the interaction effects are not usually relevant in the calculation, being sufficient to consider constant load models. However, for isostatic decks with short spans (10 m - 30 m), significant resonant effects appear with high accelerations, and often these constant load models yield results above the design limits.

With the vehicle-structure interaction models an effective reduction of these results may be obtained. The problem with these models is that they are often excessively complex for their application in project stages. The work described in section 6 quantifies numerically the reduction obtained in the dynamic response of isostatic bridges as a result of the application of the models explained in section 2.3.

The calculation procedures included in the more recent engineering design codes are simple methods applicable for isostatic structures, such as simply supported bridges, for which in practice a single mode of vibration may be considered in the response. For these cases a direct dynamic calculation may be avoided, through the use of analytic envelopes for such effects (section 2.4). However, for statically redundant structures, such as continuous deck viaducts, a direct dynamic analysis is needed, as the response includes a contribution from several vibration modes.

The railway underpasses (portal frames, vaults) are also in this latter category, as in general the deck has a statically redundant support, hence the more simplified methods for evaluation of dynamic effects are not applicable. Other aspect which complicate a correct model of the dynamic response are the possible earth cover between the deck and the ballast bed, the vibration transmitted to the earth fill in contact with the lateral walls or piers, etc. All the above

contributes to an undesirable paradox: the simplest structures, for which in practise significant resonance effects have not been observed, are the ones which a priori require a greater effort for calculation and correct evaluation of dynamic effects. The analysis of the response of portal frame underpasses is described in section 7, proposing a calculation method which adequately covers the observed dynamic response and may be evaluated through the simplified procedures for isostatic beams.

A code of good practise before performing a final time integration is to perform a sensitivity analysis with regard to the time-step. Some such considerations are discussed in general terms in section 8, regarding the recommendations which appear both in engineering codes and in the technical literature.

A simplified method for evaluating torsion effects in dynamic studies of railway bridges is proposed in [8]. According to this method, a conservative envelope results from linear superposition of the effects associated to bending and torsion, each of them analysed separately. In section 9 we present an evaluation of the fit of this simplified method.

In the first part of this paper we present a description of the basic features of the calculation methods available for dynamic analysis of railway bridges subject to traffic loads. Following a summary of the methods prescribed in the new drafts of codes IAPF and Eurocode 1 is done. Finally some research results for specific problems obtained by our group are presented.

2 ANALYSIS METHODS

2.1 Impact factor Φ

The basic method followed up to now in the existing engineering codes for design of railway bridges has been that of the impact factor, generally called Φ . As has been previously discussed in section 1, such coefficient represents the dynamic effect of (single) moving loads, but not resonance.

The general expressions contained in the codes attempt to offer simple formulae which serve as an envelope for the range of train speeds, vibration frequencies of structures and train types.

2.1.1 Code IAPF-75

The existing Spanish code IAPF-75 [19] defines a “dynamic increment”, expressed as a percentage (%), for which it prescribes the following values, for simply supported decks:

$$I = \begin{cases} 0.33v, & \text{span } L < 6 \text{ m;} \\ \frac{114\sqrt{L}}{3.10 - 1.76\sqrt{L} + L}, & \text{span } L \geq 6 \text{ m.} \end{cases}$$

In this expression v is the train speed in km/h. The dynamic increment so defined gives rise to an impact factor $\Phi = 1 + I/100$ which is applied to the static effects of certain nominal train types, called “train type A” and “train type B”, which include point loads of 30 t and distributed loads of up to 12 t/m.

For continuous beams the proposed expression is

$$I = 65 \frac{\mu}{1 - \mu + \mu^2}; \quad \mu = \frac{vT}{2L}, \quad (1)$$

where T is the fundamental period of vibration. In this formula the train speed v must be expressed in m/s (there is an typographical error in the published code, where it says erroneously km/h).

The range of speeds for application of this coefficient is $v \leq 200$ km/h.

2.1.2 Code UIC-776-1R

In this code [18], published after the Spanish proposal of IAPF-75, an impact factor is proposed associated to the static effects of a nominal train type (UIC71), which has been adopted by the majority of codes in the different countries, as well as in the Eurocode currently in effect [21]. The train type (UIC71) considers concentrated loads of 250 kN, as well as distributed loads of 80 kN/m. Note that the values of this train type are smaller than the one defined by the Spanish code IAPF-75 [19], detailed in the previous subsection.

The impact factor, for well maintained tracks, and for applying to bending moments, is

$$\Phi_2 = \frac{1.44}{\sqrt{L_\Phi} - 0.2} + 0.82; \quad \Phi_2 \geq 1. \quad (2)$$

The equivalent span L_Φ coincides with the real one for a simply supported isostatic element, and an equivalence table is provided for other structural types.

This impact factor is applied to the static values of the effects obtained from train UIC71:

$$\Phi S_{\text{est,tipo}} \geq S_{\text{din,real}}, \quad (3)$$

where $S_{\text{est,tipo}}$ and $S_{\text{din,real}}$ are, respectively, the effects corresponding to the nominal train type under static conditions and to each real train under dynamic conditions.

The above value of Φ results from obtaining a dynamic envelope for all the real trains. The interpretation of this envelope is as follows:^o

$$\Phi S_{\text{est,tipo}} \geq (1 + \varphi' + \varphi'') S_{\text{est,real}}. \quad (4)$$

It's important to remark that impact factor Φ is applied to the effects obtained for the nominal train type, whereas the factors $(1 + \varphi' + \varphi'')$ are applied to the real trains, normally much lighter than the nominal train type.

The value of φ' corresponds to the dynamic increment itself, for the train on an ideal track without irregularities, and is expressed as:

$$\varphi' = \frac{K}{1 - K + K^4}; \quad K = \frac{v}{2L_\Phi f_0}. \quad (5)$$

(Note that in this formula K coincides with parameter μ of expression (1) within code IAPF-75, and that both expressions are very similar, with the only difference of the last term in the denominator, in one case a fourth power and a second power in the other).

The value of φ'' arises from the effect of track irregularities, with the following value:

$$\varphi'' = a \left[0.56e^{-\left(\frac{L\Phi}{10}\right)^2} + 0.50 \left(\frac{f_0 L\Phi}{80} - 1 \right) e^{-\left(\frac{L\Phi}{20}\right)^2} \right] \quad (6)$$

where $a = \min\left(\frac{v}{22}, 1\right)$, with velocity v expressed in m/s.

The application of impact factor Φ is subject to some conditions, which ensure that it corresponds to the real scenarios of bridges and trains for which it was formulated and its validity checked. Specifically, the fundamental frequency of vibration of the bridge must be within the limits of a *band*, defined with respect to the span of the bridge in figure 7.

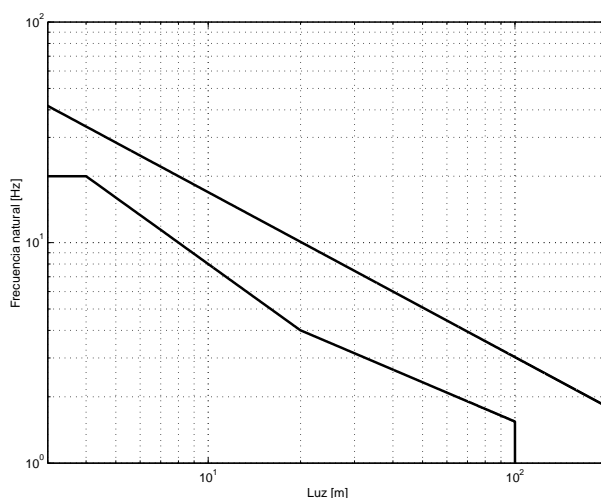


Figure 7. Bounds for the natural frequency f_0 in Hz, in relation to the span of the element, for application of impact coefficient Φ

The coefficient Φ so defined does not take into account resonant effects. With the object of avoiding this possibility, most engineering design codes that include it (e.g. [21]) limit its use to velocities $v \leq 200$ km/h.

2.2 Dynamic analysis with moving loads

As has been said above, the method of the impact factor, as a counterpart to its simplicity, has a number of limitations. The main one is that, not taking into account resonance, it is not applicable for high speeds ($v > 200$ km/h). In these cases one may perform a dynamic analysis with moving loads.

These methods are based on time integration of the dynamic equations for the structure, when subject to a series of moving loads of fixed values, representative of each axle of a given train. The model for the structure may be analysed either through an integration of the complete system with N degrees of freedom, or through a reduction of degrees of freedom from a modal analysis which reduces substantially the number of equations to integrate. This modal

reduction may be performed through an approximate numerical procedure to obtain the eigenmodes of vibration, a capability which is provided by the majority of finite element programs. Alternatively this may be achieved through an analytical (closed form) calculation, for certain cases of simple structures.

2.2.1 Analytical methods

The classical problem of a *simply supported isostatic bridge* may be treated through the *exact* eigen modes of vibration, which correspond to the hypotheses of the Bernoulli beam [9], for which the modal shapes are $\phi_n(x) = \sin(n\pi x/l)$ and the associated eigenfrequencies $\omega_n = (n\pi)^2 \sqrt{EI/(\bar{m}l^4)}$. Figure 8 shows the first three vibration modes for this case. Generally,

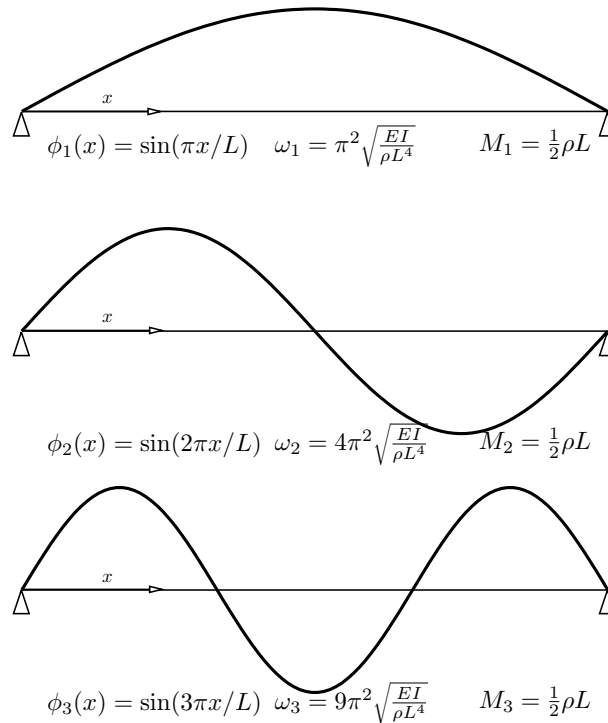


Figure 8. First three vibration modes for an isostatic simply supported beam

for an isostatic case, it is enough to consider a single vibration mode; this way the problem is reduced to a dynamic equation with one degree of freedom, whose solution and interpretation is much simpler than other cases with multiple degrees of freedom.

For more complex *statically redundant structures* it is not possible in general to perform an analytical extraction of vibration modes and frequencies. Nevertheless, analytical closed-form solutions may be obtained for some specific cases, such as (intraslational) portal frames and continuous beams with two or three spans [15]. For rectangular portal frames the procedure detailed in [12] is slightly more complex than for the simply supported bridge. For example, the two first vibration modes are shown in figure 9. The expression for the frequency associated

to the first mode is defined through a parameter b with the equation:

$$\omega_1 = \left(\frac{b}{l_d}\right)^2 \sqrt{\frac{EI}{\bar{m}_d}} \quad (7)$$

where l_d is the span of the deck, $E_d I_d$ its bending stiffness and \bar{m}_d the mass per unit length. Parameter b is obtained as the solution of the following nonlinear equation:

$$\frac{k_p(1 - \cosh(k_i b) \cos(k_i b))}{\cosh(k_i b) \sin(k_i b) - \sinh(k_i b) \cos(k_i b)} + \frac{1 - \cosh b \cos b}{(\cosh b + 1) \sin b - (\cos b + 1) \sinh b} = 0 \quad (8)$$

being:

$$k_p = \sqrt[4]{\frac{I_d^3 \bar{m}_d}{I_h^3 \bar{m}_h}} \quad k_i = \frac{l_h}{l_d} \sqrt[4]{\frac{I_d \bar{m}_h}{I_h \bar{m}_d}} \quad (9)$$

In this equation, subindex h refers to the piers or vertical walls of the portal frame.

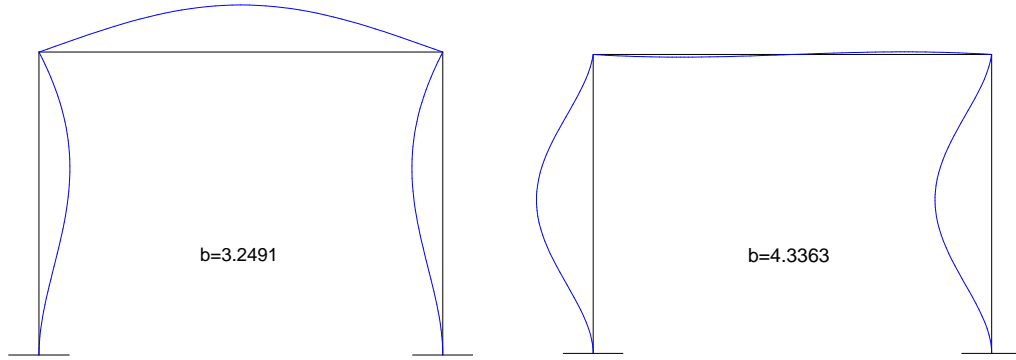


Figure 9. Two first vibration modes of a portal frame corresponding to an underpass of a new high speed railway line, with the value of parameter b for calculation of eigenfrequencies through equation (7)

Once the vibration modes are known, it is necessary to integrate the dynamic equations. For this, the basic solution is the response of the structure to a single moving load (figure 10). Consider a continuous beam of length l , being $\phi_i(x)$, M_i and ω_i respectively the modal shape, the modal mass and the eigen frequency of the i -th mode. The differential equation for a point load F crossing the beam at a constant speed v is:

$$M_i \ddot{y}_i + 2\zeta_i \omega_i M_i \dot{y}_i + \omega_i^2 M_i y_i = F \langle \phi_i(vt) \rangle \quad (10)$$

where y_i is the modal amplitude of the i -th mode, ζ_i the damping fraction with respect to the critical value, and $\langle \phi(\bullet) \rangle$ represents a bracket notation with the following meaning:

$$\langle \phi(x) \rangle = \begin{cases} \phi(x) & \text{if } 0 < x < l \\ 0 & \text{otherwise.} \end{cases} \quad (11)$$

After obtaining the response for a single moving load, the response for a load train may be assembled as the superposition of the responses for the point loads F_k (figure 11). The

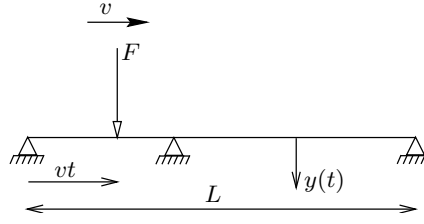


Figure 10. Response for a single moving load

differential equation corresponding to mode i is in this case:

$$M_i \ddot{y}_i + 2\zeta_i \omega_i M_i \dot{y}_i + \omega_i^2 M_i y_i = \sum_{k=1}^{n_{\text{ejes}}} F_k \langle \phi_i(vt - d_k) \rangle. \quad (12)$$

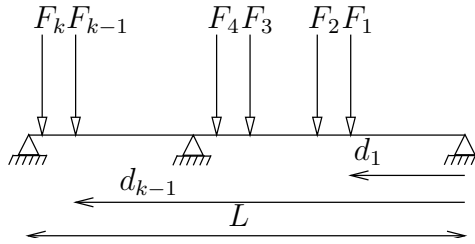


Figure 11. Response for a load train

2.2.2 Finite element methods

Dynamic analysis of railway bridges based on moving load models may also be performed through finite element methods. These methods are applicable generally to arbitrary structures, and may include if necessary nonlinear effects.

A spatial discretisation of the structure is performed into subdomains called finite elements, obtaining an approximate model with a discrete number of degrees of freedom N , followed by a time discretisation in time-steps. The analysis may then be carried out by direct time integration of the complete model, or alternatively through modal reduction. In both cases the basic problem to be solved is the system of differential equations:

$$\mathbf{M} \ddot{\mathbf{d}} + \mathbf{C} \dot{\mathbf{d}} + \mathbf{K} \mathbf{d} = \mathbf{f}, \quad (13)$$

where \mathbf{M} is the mass matrix, \mathbf{C} the damping matrix, \mathbf{K} the stiffness matrix, \mathbf{f} the external load vector, and \mathbf{d} the vector of (unknown) nodal displacements.

By means of the direct integration of the model, the complete system (13) of N degrees of freedom would be solved for each time step; the equations are generally coupled, and therefore must be solved simultaneously. This procedure is also valid when nonlinear effects are

to be included in the response; in this case the elastic internal forces and viscous damping forces from the previous expression should be replaced by a general term (nonlinear) of the type $\mathbf{F}^{\text{int}}(\mathbf{d}, \dot{\mathbf{d}}, \dots)$.

If the structural behaviour is linear, a modal analysis may be performed, with a remarkable reduction of degrees of freedom. In a first stage, the eigenvalue problem is solved, obtaining numerically the more significant n eigenfrequencies and associated normal vibration modes (generally $n \ll N$). Afterwards, these vibration modes are integrated in time. This way the equations become uncoupled, and the modal response of each mode is reduced to the dynamic equation of a system with a single degree of freedom [9].

The simplest procedure to model load trains is applying load histories in each node. At a certain time-step, a load is assigned to each node if the load axis is above an element that contains the node. The magnitude of the nodal load depends on the distance from the axis to the node. This procedure is outlined in the figure 12 for a generic node A .

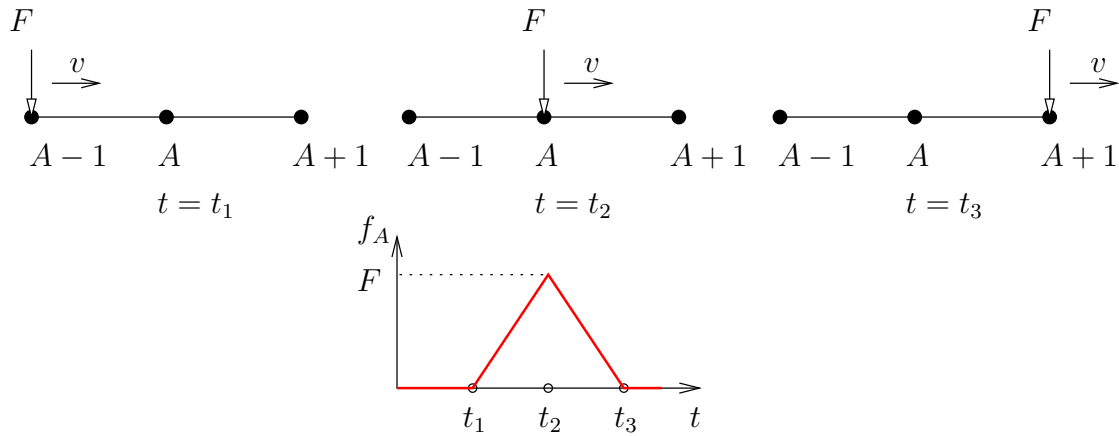


Figure 12. Nodal force definition for node A for a single moving load F .

This scheme is applied to the real trains defined in code [13], and has been implemented in finite element program FEAP [10]. The results described in the report [12] have been obtained with this methodology and time integration of the vibration modes.

2.3 Dynamic analysis with vehicle-structure interaction

The dynamic analysis with vehicle-structure interaction consists, like the analysis with moving loads, of a direct time integration of the dynamic equations of the structure jointly with the vehicle vibration due to the its own suspension; thus, the axis loads do not have in fact a fixed value during the crossing of the bridge.

This type of model represents, in the most general case (figure 13), the primary suspension of each axle with the values of stiffness and damping (K_p, c_p), the secondary suspension, with the corresponding values of bogie stiffness and damping, (K_s, c_s), the non-suspended mass, corresponding to the nominal mass of the axis of the wheel (m_w), the length, mass and moment

of inertia of the bogie (L_B, m_b, j_b), the suspended mass and moment of inertia that corresponds to the box of the vehicle (M, j) and the geometry of the vehicle: total length (L), distance between the centre of gravity of the box of the vehicle and the front and rear axis (d_{Bd}, d_{bt}), and the distance between the axis of a bogie (d_{eB}). For those vehicles whose guidance system is not

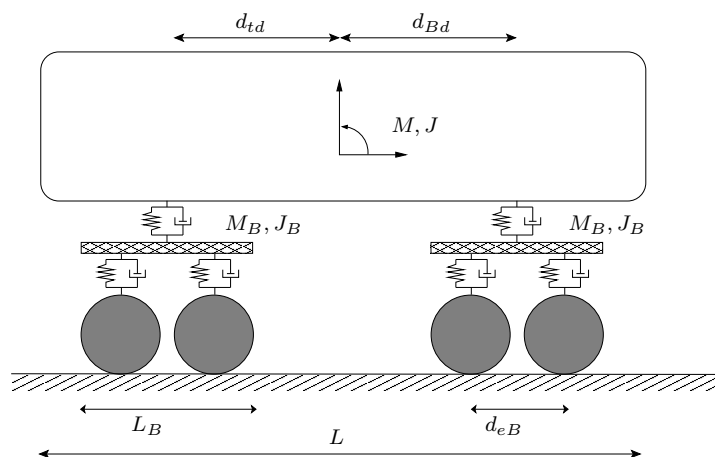


Figure 13. Complete vehicle–structure interaction model.

accomplished by bogies, the previous scheme should be adapted to the particular configuration of the axes and the suspension system, with equivalent level of detail.

The level of detail in the interaction models described above is not always necessary. *Simplified models* of vehicle–structure interaction may also be employed, where each axis suspension is modelled independently from the others, without considering the effect of connection with the vehicle box. This way, they consider (figure 14) primary axis suspension with its corresponding values of stiffness and damping (K_p, c_p), non-suspended mass corresponding to the nominal mass of axis wheel plus the proportional part of totally suspended mass (vehicle box) (m_{ns})¹, and the suspended mass; in this case, its value is equivalent to the proportional part of the bogie mass (m_s). Another variant equivalent to this model is proposed in the new UIC 776-2 leaflet[8]; this model is represented in the figure 14.

It is important to mention that in simplified interaction models each axis is independent from the rest —this means that there is no interaction between the axes of a same vehicle—, whereas in the complete models there exists certain interaction among them, because the model represents the complete vehicle box.

Proposed interaction model.— This model has been implemented in [5], a computer application which has been used for the research work reported in this article. It considers a train of k loads, representing each axle according to a simplified vehicle–structure interaction model (figure 15).

¹Remark that even though m_{ns} (simplified model) and m_w (complete model) are referred by the same words

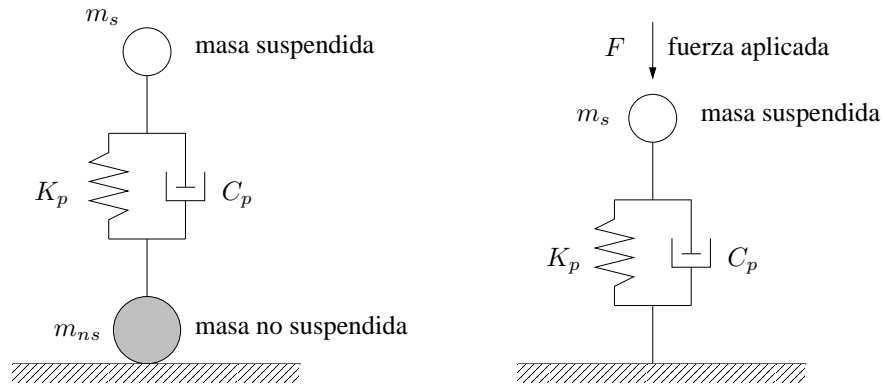


Figure 14. Simplified vehicle–structure interaction model (left). Variant to the proposed model in the UIC-776-2 code [8] (right)

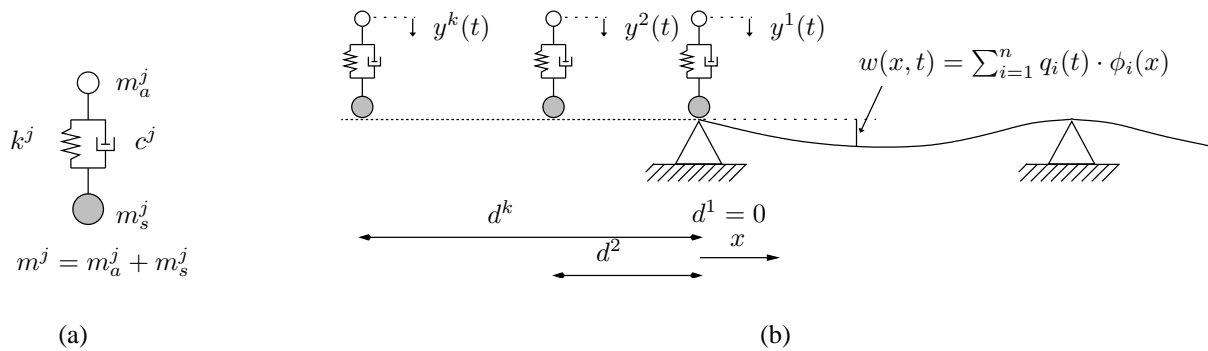


Figure 15. Crossing of a train of loads, according to the vehicle–structure interaction simplified model: a) interaction element; b) geometric definition of variables

When analysing for a load train, the number of differential equations to solve is increased; for a single axle, the number of equations considered are vibration modes n plus the corresponding one to the mechanical system of the simplified interaction element, altogether $n + 1$. Supposing a group of k loads, a system of $n + k$ differential equations is obtained.

The equations corresponding to the vibration modes of the bridge differ only in the term which represents of the modal load for each time step, for which it will be necessary to calculate the axles which are on the bridge and the value of the amplitude corresponding to the position of these.

For a general case, the following equations are obtained for the model:

- For each vibration mode ($i = 1 \dots n$):

$$M_i \ddot{q}_i + C_i \dot{q}_i + K_i q_i = \sum_{j=1}^k \langle \phi_i(d_{rel}^j) \rangle (g m^j + m_a^j \ddot{y}^j) \quad (14)$$

- For each interaction element ($j = 1 \dots k$):

$$m_a^j \ddot{y}^j + k^j [y^j - \sum_{i=1}^n q_i \langle \phi_i(d_{rel}^j) \rangle] + c^j \left[\dot{y}^j - \sum_{i=1}^n \dot{q}_i \langle \phi_i(d_{rel}^j) \rangle - \sum_{i=1}^n q_i v \langle \phi_i'(d_{rel}^j) \rangle \right] = 0 \quad (15)$$

In equations (14) and (15) the notation $\langle \phi(\bullet) \rangle$ has been employed, defined previously in equation (11). Additionally, d_{rel}^j represents the relative position on the bridge for each element j . Considering the initial time $t = 0$ when the head of the train enters the bridge ($x = 0$), d_{rel}^j is obtained as:

$$d_{rel}^j = vt - d^j \quad (16)$$

Considering the nature of the resulting equations, (system of linear second order ordinary differential equations), the trapezoidal rule is recommended for its integration. The trapezoidal rule is a member of the β -Newmark family, defined by $\beta = 1/4$ and $\gamma = 1/2$. In [4] this and some other aspects of the implementation of integration models in dynamic analysis of railroad bridges are discussed.

2.4 Models based on harmonic series

This type of models avoid a direct dynamic analysis by time integration. In counterpart its application is limited to *simply supported bridges*, which can be represented dynamically by means of a single harmonic vibration mode.

The different models of this type all develop the response as a combination of harmonic series, and establish an upper bound of this sum. The different available models of this type also introduce another concept, of special relevance in the intuitive interpretation of the response: *dynamic signature* associated to a given train of loads.

—non-suspended mass—, their values are obtained from different concepts.

As its name indicates, the *dynamic signature* of a train may be understood as a function which characterises its aggressiveness in relation to the dynamic effects produced in a railroad bridge. The models of this type proposed are:

- **DER:** Based on the Decomposition of the Resonance Excitation.
- **LIR:** Simplified method based on the Residual Influence Line.
- **IDP:** Simplified method based on the Proportional Dynamic signature.

The methods DER and LIR originate within the group of experts of the European Railway Research Institute (ERRI), in the D214 committee on railroad bridges for speeds greater than 200 km/h [7]. The method IDP has been developed in [4] and [23].

All these models have their application limited to isostatic *bridges*; it is considered that the dynamic response of this kind of bridges may be represented by only the first bending vibration mode of the structure. In addition, the first mode shape is an harmonic function, a fact which facilitates the analytical development of the series.

The DER method originates from the decomposition of the dynamic response of the bridge in Fourier series, and focuses on the study of the term that corresponds to the resonance in frequencies. This way, it obtains an upper bound of the maximum acceleration in the centre of the beam as a product of two functions: the first one characterises the response of the bridge and the second one is the so called dynamic signature of the train.

The mathematical development of LIR method is based on the analysis of the free vibrations produced after each individual single load crosses a simply supported bridge, according to the dynamic analysis of a beam put under the action of successive single loads.

Proposal IDP is centred in the study of residual maximum acceleration of each single load, obtaining a slightly improved interpretation of the dynamic signature than LIR method. In particular, it considers the damping that takes place since an axis enters the bridge until it leaves it with a state of residual vibration.

All these methods end up limiting analytically the maximum dynamic response, in terms of acceleration or displacement at a given point, as a product of three terms. In this triple product the contribution from the structure and from the aggressiveness of the circulating train are clearly differentiated .

Take as an example the LIR method proposed for the maximum acceleration. This value at the centre of the span, Γ , is obtained as a product of the following factors:

$$\Gamma = C_{\text{accel}} \cdot A(K) \cdot G(\lambda), \quad (17)$$

where $C_{\text{accel}} = 1/M$ is a constant (the inverse of the total mass of the isostatic bridge), $\lambda = v/f_0$ (wavelength), with v the circulation speed and f_0 the eigen frequency (Hertz) of the first vibration mode, and $K = \lambda/(2l)$, being l the span of the simply supported bridge. The

other terms have the following definition:

$$A(K) = \frac{K}{1 - K^2} \sqrt{e^{-2\zeta \frac{\pi}{K}} + 1 + 2 \cos\left(\frac{\pi}{K}\right) e^{-\zeta \frac{\pi}{K}}} \quad (18)$$

$$G(\lambda) = \max_{i=1}^N \sqrt{\left[\sum_{x_1}^{x_i} F_i \cos(2\pi\delta_i) e^{-2\pi\zeta\delta_i} \right]^2 + \left[\sum_{x_1}^{x_i} F_i \sin(2\pi\delta_i) e^{-2\pi\zeta\delta_i} \right]^2} \quad (19)$$

In these expressions ζ is the damping rate, x_i are the distances of each one of the N load axes F_i to the first axis of the composition, and $\delta_i = (x_i - x_1)/\lambda$.

The term $G(\lambda)$ (equation (19)) is the previously *dynamic signature* referred to above. It depends only on the distribution of the train axles and the damping rates. Each load train has its own dynamic signature, which is independent of the mechanical characteristics of the bridges. The figure 16 represents the dynamic signature of train ICE2, for different values of damping.

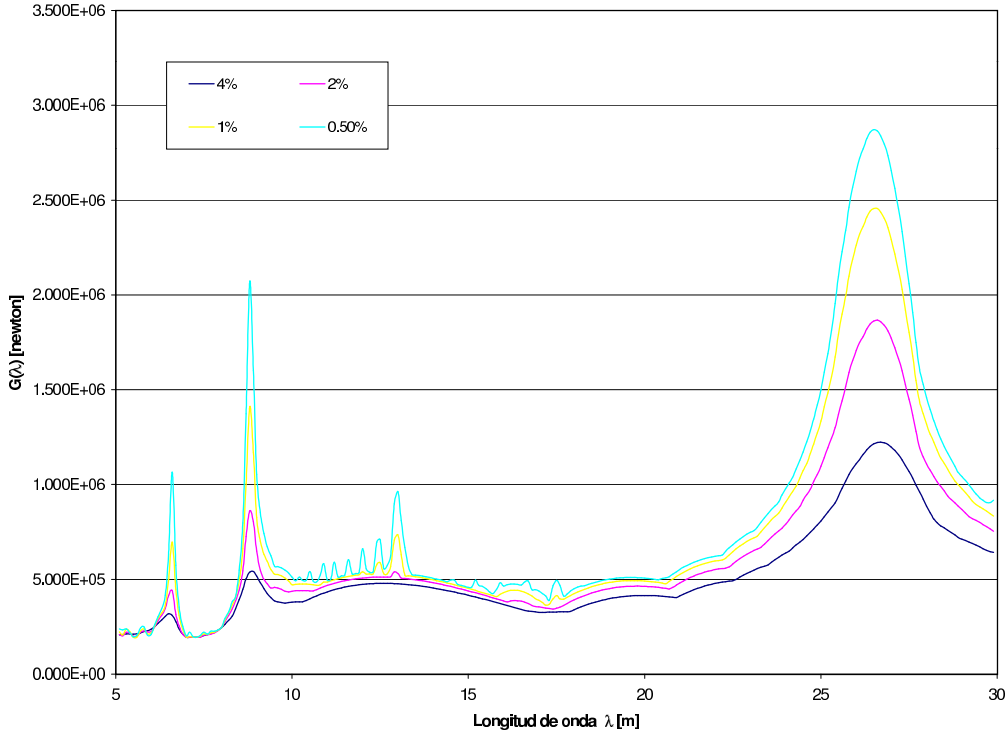


Figure 16. *Dynamic signature of train ICE2 with different damping rates.*

$A(K)$ is a function determined for each particular bridge; it depends on the span of the bridge l , its natural frequency (f_0), damping (ζ) and the range of speeds of circulation (v) under study. This function of parameter r is called the bridge *dynamic influence line*.

Taking the three parameters considered, neither C_{acel} nor $A(K)$ depend on the characteristics of the train. Separating the contributions from the bridge and those from the train ($G(\lambda)$, dynamic signature), it is possible to determine easily the critical parameters of span and wavelength

(proportional to the train circulation speed, v) that maximise the response of the bridge.

3 HIGH-SPEED REAL TRAINS AND INTEROPERABILITY

A characteristic of resonance is that its occurrence for a certain bridge depends mainly on the circulation speed (critical resonance speeds) and the type of train.

With relation to the circulation speeds, dynamic analysis sweeps should be undertaken as a general rule with speed increments of 2, 5 or 5 km/h (never greater than 10 km/h). For the frequent speeds of circulation, this sweep should be refined.

With respect to the train types, although initially a certain rail line may be operated by one or more given high speed train types, it seems clear that the trains which can circulate should not be restricted. This would allow to modify the operation conditions and the high speed trains that may circulate; furthermore, it opens the possibility of interconnection with other European high speed lines, permitting all the European trains to circulate along the lines that have been developed with *interoperability criteria*.

In order to guarantee this capability, the dynamic analysis of a certain bridge or viaduct should be performed made for all the current and future types of *real trains*.

Another possibility that could be more advantageous is to establish interoperability conditions which cover the characteristics of the existing real trains and the anticipated evolutions; thus, these criteria could be imposed to the new trains to be developed from now on. Given these conditions, it should be able to establish *universal trains* which are dynamic envelopes of the effects of the possible real trains.

3.1 Real trains

European high speed real trains are of three different types (figure 17):

1. *Conventional trains*: each passenger car has two bogies, with two axes each one. Of this type are the trains Ice2, Etr-y, Virgin.
2. *Articulated trains*: the passengers cars have one bogie of two axles in each end, shared with the adjacent car and centred under the joint between them. This type includes the Thalys and Eurostar. The Ave train is similar to the Thalys.
3. *Regular trains*: the passenger cars are also articulated, but this joint is not supported on a shared bogie, but on a single axis between them. It is the case of Talgo AV.

The new code IAPF [13] details the compositions of each one of these types of trains. Knowing these, it is possible to obtain the dynamic signature of each one of the real trains (figure 18), and from these, to obtain a signature envelope (figure 19). This envelope could be used in the dynamic analysis, with a clear benefit as to simplification of calculations.

3.2 High-speed interoperability load model (HSLM)

The following objections may be raised with regards to the previous description of a reference envelope study (figure 19):

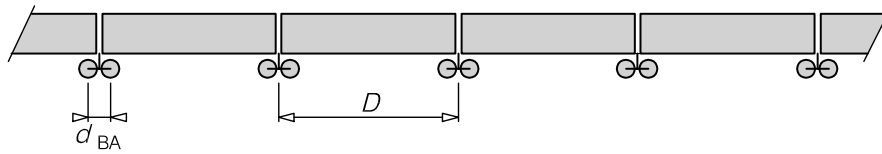


Figure E1 - Articulated train

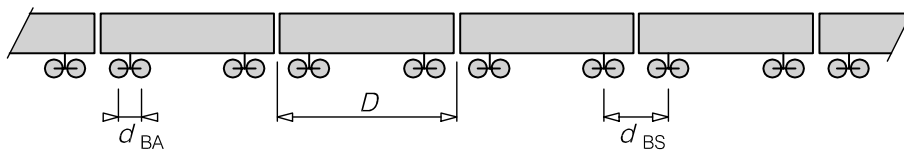


Figure E2 - Conventional train

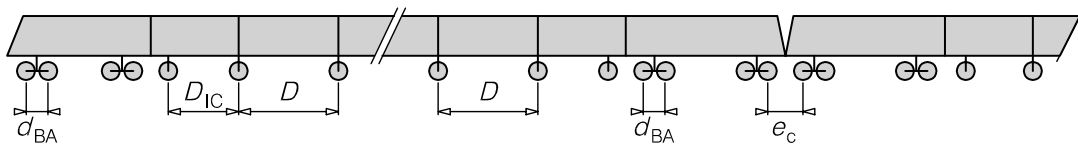


Figure E3 - Regular train

Figure 17. *Different types of high-speed trains, according to Eurocode 1 [22]*

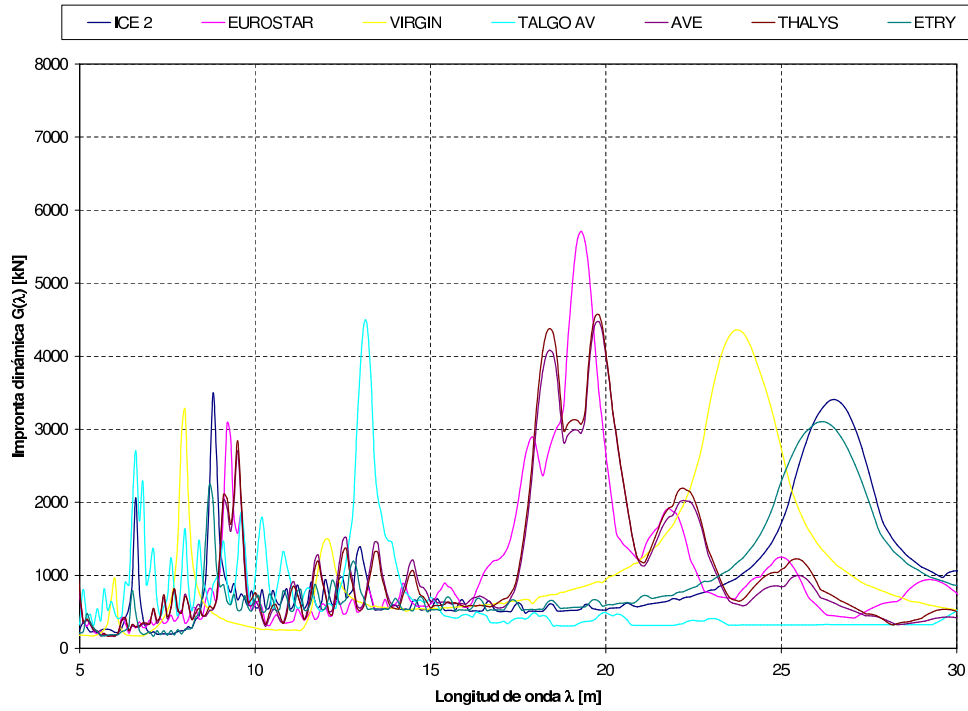


Figure 18. Dynamic signature (undamped) for European high-speed trains.

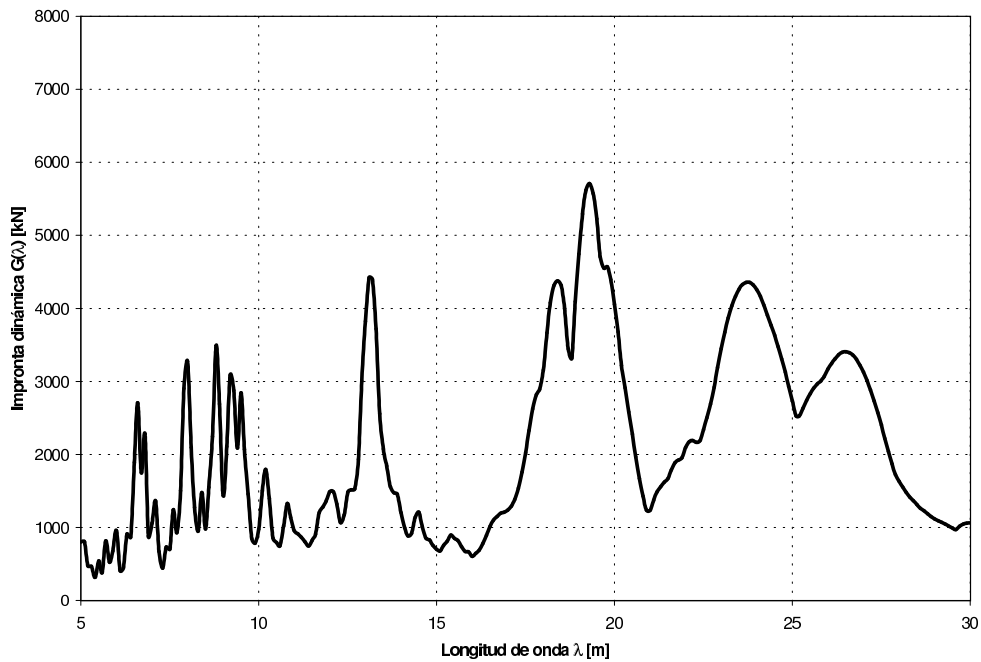


Figure 19. Envelope of the dynamic signatures of the European high-speed trains; structural damping rate $\zeta = 0\%$

- **Generality:** The obtained envelope is not general enough as to be proposed to rule out designs in future trains: a small variation of parameters of a present train — say the length of the vehicles, the nominal value of the load by axis or the distance of connections— could modify its original signature, and it would subsequently not be covered by the envelope. Therefore, it is possible to conclude that adopting the reference envelope thus defined is too restrictive;
- **Analysis procedure:** The envelope of the dynamic signature of the real trains defines a reference aggressiveness for the construction of new trains, but it does not propose a specific method of calculation associated to this envelope. This fact represents a drawback in the envelope proposal, since it does not solve one of the principles of the rail lines interoperability: to provide a simplified methodology of analysis associated to the reference dynamic envelope.

In order to allow a more general analysis method, not subjected to these disadvantages, committee ERRI D214 [26] has defined analytically a family of universal trains whose dynamic effects on the structures covers any real train that circulates at the present (and future) time. This family —called UNIV-A— has the characteristics gathered in the table 1.

	UNIV-A
Type	articulated
Total length	$\simeq 400$ m
Cars lengths D	from 18 to 27 m
Axis load	170 kN
Distance between axes of the same bogie	2.5 m
Head and tail locomotives	yes

Table 1. *Characteristics of UNIV-A universal trains. Font: Technical report of ERRI D214 Committee [26]*

From its definition, the universal trains constitute a family that depends on the vehicle length D . That is, each value of D from 1 which is within the range proposed in the table corresponds to a member of this family. The set of trains that are generated by variation of parameter D constitutes the envelope of the universal trains UNIV-A.

The envelope of the UNIV-A trains is shown in figure 20, based on the adopted discretisation of D . Further, figure 21 represents, for each value of the wavelength λ , the length of the car D that corresponds to the critical signature in that point, the one that defines the value of the envelope.

3.2.1 UNIV-A and the Talgo AV

In order to obtain an envelope dynamic signatures which would cover the dynamic effects of all the real trains, [26] proposed the envelope of the UNIV-A trains; however, figure 22 shows that this signature does not cover the regular train Talgo AV for wavelengths smaller than 15 meters

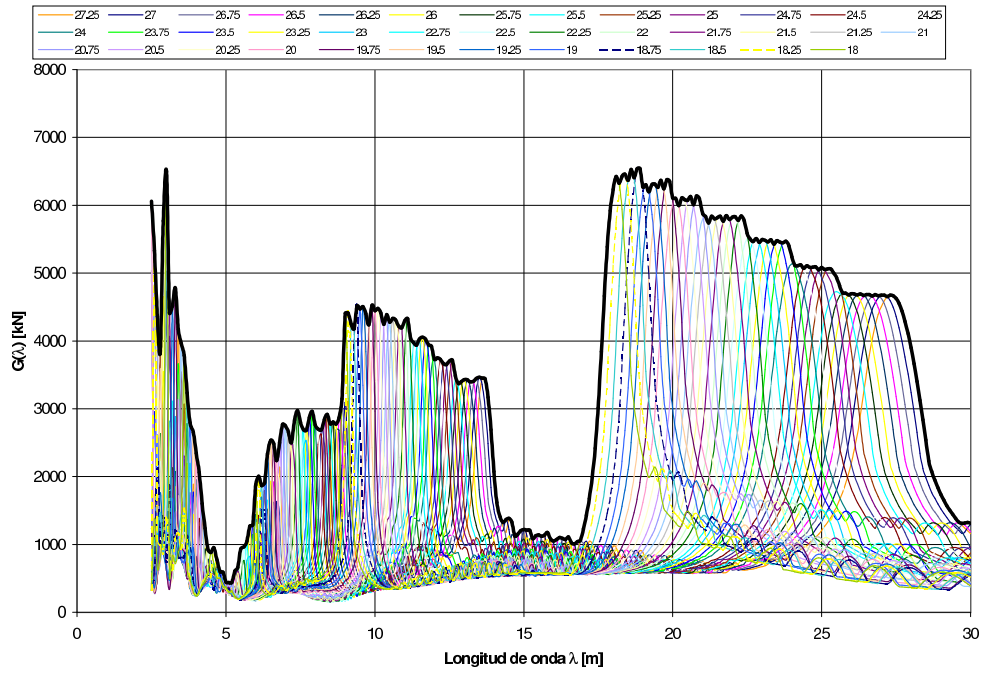


Figure 20. Dynamic signature of the UNIV-A trains: obtaining the envelope for different vehicle lengths D .

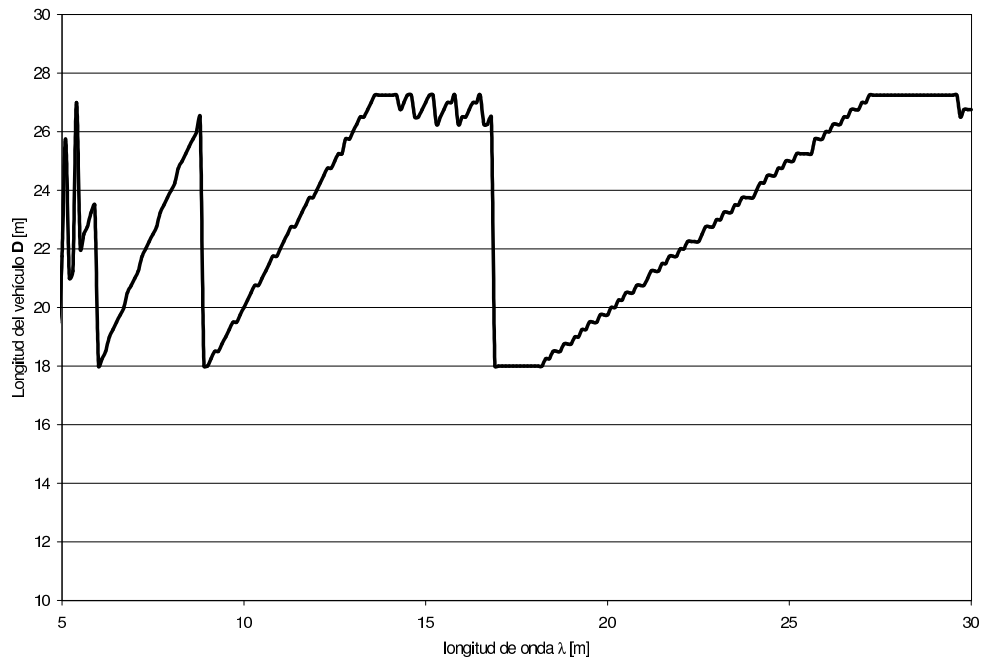


Figure 21. Relation $\lambda - D$ for the UNIV-A trains envelope. Damping rate $\zeta = 0\%$.

($\lambda < 15$). A similar result would be obtained for other real regular trains, since Talgo AV is a representative example of such trains.

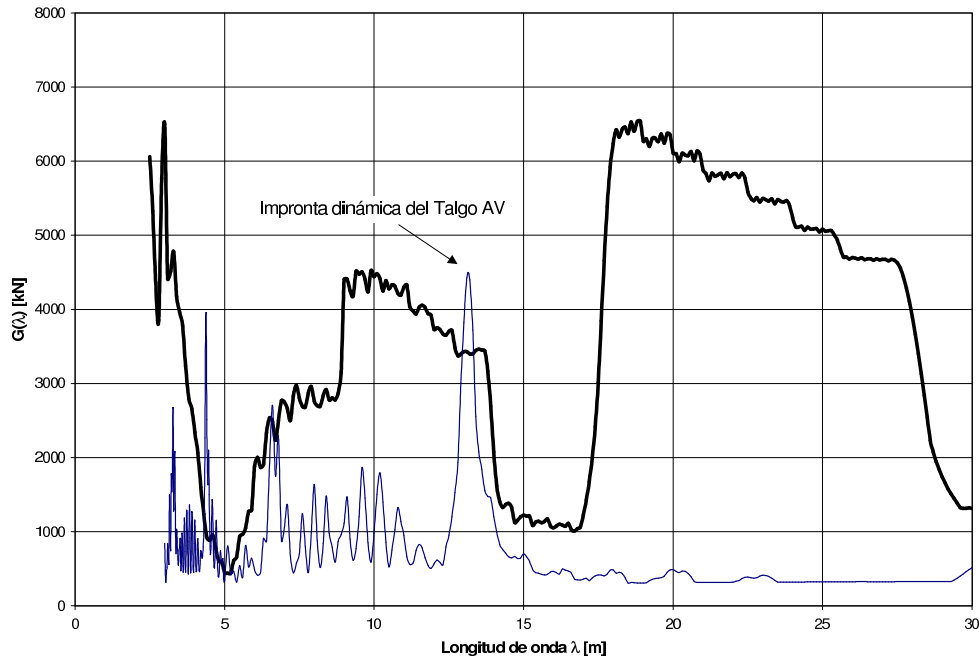


Figure 22. *Dynamic signatures of the UNIV-A and the Talgo AV envelope.*

In order to complete the envelope with regard to regular trains, a study was carried out which concluded with a proposal to introduce some modifications to the definition of the UNIV-A family, so that for short wavelengths certain additional trains would be considered to cover the envelope of such a regular trains like the Talgo AV [27]. These proposals were taken into account and have given rise to the final definition of the High Speed Load Model (HSLM), family of trains contained in the last and recent version (final draft) of Eurocode 1 of actions in railroad bridges [22].

4 DYNAMIC ANALYSIS IN THE SPANISH CODE FOR ACTIONS ON RAILWAY BRIDGES (INSTRUCCIÓN DE ACCIONES EN PUENTES DE FERROCARRIL, IAPF)

Taking into account the above discussion the basic provisions for considering the dynamic response of bridges in the new draft code on actions in railroad bridges (IAPF) are defined next. [13].

4.1 Vertical traffic load model

The load model adopted for vertical actions of railway traffic is the UIC71 —also named LM71—, which constitutes a standard in practically all the countries of our surroundings and in

Eurocode 1 [21, 22]. Further, the use of a *classification factor* $\alpha = 1.21$ for international and RENFE rail lines is prescribed. For metric (narrow gauge) rail lines, $\alpha = 0.91$ has been adopted.

This factor α in fact increases the loads of model LM71, the concentrated loads of 250 kN become 302 kN, and the distributed loads of 80 kN/m become 96.8 kN/m. These values are similar to those considered until now in the existing Spanish code for design of railroad bridges IAPF-75 [19] (see also section 2.1.1).

The main reasons that have led the technical commission to prescribe the factor $\alpha = 1.21$ are the following:

- it has been verified that it leads to broadly equivalent effects as the existing load models from the IAPF-75 [19];
- it preserves the capacity of the railway network; otherwise, using the new model LM71 without α the capacity would have been reduced and the network performance decreased;
- the cost of introducing this factor $\alpha = 1.21$ has been evaluated, resulting in extremely small increases in relation to the complete structure.
- With this increase it suffices consider only one load model, additional trains SW/0 (for continuous bridges), SW/2 (heavy traffic) [22] are not necessary;
- it allows to extend the validity of the impact factor Φ up to 220 km/h, as the margin obtained by multiplying the static loads of the load model, with respect to the static loads of the real trains is greater and can accommodate moderate dynamic increases.

4.2 Envelope dynamic factor Φ

The basic result of the evaluation of the dynamic effects is the impact factor Φ . It is defined as the envelope of the effect under consideration:

$$\Phi = \max \frac{S_{\text{din,real}}}{S_{\text{est,tipo}}}$$

The relevance of this definition is that the same concept of envelope impact factor is applicable also to the cases where a dynamic analysis is carried out (here $S_{\text{din,real}}$ will be calculated), whose result will also be synthesised as an impact factor.

The direct formula of the impact factor is the same one included in Eurocode which originates from [18],

$$\Phi = \frac{1.44}{\sqrt{L_{\Phi}} - 0.2} + 0.82,$$

as was described in section 2.1.2 (equation (2) for tracks with good maintenance). The limitations for using this factor are:

1. $v \leq 220$ km/h;
2. *conventional* structure (that is, included in the corresponding tables for equivalent span L_{Φ});
3. fundamental frequency of the bridge f_0 within the limits of figure 7.

In any other case, when one or more of the previous conditions are not fulfilled, it will be necessary to perform a more detailed dynamic analysis; the methodology is described in appendix B of [13]. These procedures are summarised in the following subsections.

4.3 Dynamic analysis procedures

For the cases when it is not possible to use the envelope factor Φ , several dynamic analysis procedures are established. These procedures (except for the one mentioned in first place, which is only applicable for $v \leq 220$ km/h) will be applied either for high speed real trains (the axle loads are defined in the code), or for the universal load model HSLM which is also included; this HSLM model allows to guarantee interoperability of the rail lines. It consists of a family of 10 HSLM-A trains, except for short spans $L < 7$ m, for which a special family HSLM-B needs to be applied.

As the critical resonance speed is unknown beforehand, the calculations need to be carried out as sweep, analysing all the cases between $v = 220$ km/h and the maximum velocity of the line, increased by factor 1.2 as a safety margin. A large number of dynamic analyses may hence be necessary, specially if it is desired to evaluate different structural hypotheses; for this reason this dynamic analysis work should be planned carefully.

1. *Real impact factor.*

This procedure boils down to evaluating more precisely the individual dynamic factors φ' and φ'' , according to the formulae (4), (5) and (6).

This method may be used for rail lines with circulation speed $v \leq 220$ km/h, when some of the other specified requirements for the factor Φ are not fulfilled, for example when the structural fundamental frequency is not within the limits of figure 7. For the analysis of factor φ' a set of trains defined specifically for speeds $v \leq 220$ km/h should be used.

2. *Dynamic signature (DER, LIR)*

This procedure allows to evaluate the dynamic effects without making a dynamic analysis with time integration, as has been described previously in section 2.4. It suffices to evaluate the expressions indicated in the formulae (17), (18), (19); even though at first sight the expressions seem somewhat complex, they can be programmed very easily in any computer spreadsheet. The analysis method is considerably simpler than a direct time dynamic analysis.

This method has the disadvantage that it is only applicable for isostatic bridges; this is an important limitation for many viaducts and real structures that do not fulfil this condition.

3. *Dynamic analysis by direct time integration with moving loads*

For the general cases of bridges which are not simply supported beams, a dynamic analysis with moving loads and the consideration of the real structural model (or modes of vibration) must be carried out. as has been already described above in section 2.2. It is possible to use either direct analytical procedures for the extraction of the vibration modes (in the simplest cases) or more general methods by means of finite elements, that allow to analyse any type of structure.

4. *Dynamic analysis by time integration with vehicle-structure interaction.*

These are the last type of models mentioned in the section 2.3. The level of effort and complexity of these models may result in an excessive complexity for ordinary project analysis. Nevertheless, they can be useful to improve the analysis either in some special project situation or as a part of a research work. It should be considered that with the interaction models the the bridge dynamic effects result generally in smaller, more realistic values, comparing to simpler moving load models, specially in short span bridges (see [11], described also in section 6; reductions of dynamic effects of up o 45% are obtained in resonance).

The additional complexity makes the work more difficult and risky for the analyst who defines the model and interprets the results, which is why these analysis must be approved by the competent authority.

In any of the previous cases, the result of the dynamic analysis may be interpreted to yield a dynamic factor (without considering track irregularities):

$$\boxed{\text{dynamic analysis}} \rightarrow (1 + \varphi') = \frac{S_{\text{din,real}}}{S_{\text{est,real}}} \quad (20)$$

Once this is calculated, the effect of the track irregularities may be added by means of factor φ'' , which should be applied to the previous factor (20) according to:

$$(1 + \varphi')(1 + \varphi''/2). \quad (21)$$

The maximum of all these values for all the possible trains and circulation speeds will constitute the impact factor Φ .

It is also convenient to remember that the dynamic analysis results must be applied to verify not only the ultimate limit states (ULS), but also the service limit states (SLS) related to deformations and maximum accelerations; this is the object of another paper in this congress [24]. In particular, one of the conditions that might be difficult to satisfy for certain low spans bridges is the limit of maximum accelerations (0, 35g for bridges with ballast bed, 0, 50g for the rest).

Finally, it is noticed that for bridges with two or more tracks, the static loads $\alpha \times \text{LM71}$ must be simultaneously applied in the two tracks. However, the absolute maximum dynamic effects should not be added for the two tracks, rather they should be combined by means of the rule of the square root of the sum of the squares. These dynamic effects are the ones included within coefficient φ' in the expression (20).

5 DYNAMIC ANALYSIS IN EUROCODE 1 (PREN 1991-2)

The last Eurocode 1 of structural actions, section 2 (actions on bridges), is the last proposal of a revision period; the last studies and analysis results arrived at by ERRI have been included, in particular those from the ERRI D214 committee for structural effects of high speed rail lines [8]. It is a very recent report, dated 10/01/2002. The more remarkable aspects follow in the next sections.

5.1 Vertical traffic load model

Load model LM71 (UIC71) is adopted, allowing also the possibility to establish by the national administration a classification factor $\times \alpha$. Additionally, trains of loads SW/0 (continuous bridges), SW/2 (heavy trains), and the train without load should be used in certain specific situations.

5.2 Conditions for dynamic analysis

These conditions are established by the Eurocode by means of a flow chart where the different possibilities are included (figure 23).

We indicate in summary the main aspects deduced from this chart:

- *Situations where only a static analysis is required, with impact factor Φ :*
 - $v \leq 200$ km/h and continuous bridge;
 - $v \leq 200$ km/h and f_0 within the limits of figure 7;
 - $v > 200$ km/h and isostatic bridge with span $L \geq 40$ m, and f_0 within the limits of figure 7;
 - $v > 200$ km/h and isostatic bridge, $f_{\text{torsion}} > 1.2f_0$, + use of tables F1/F2 tables to verify accelerations (ELS);
- *Situations where a direct dynamic analysis is necessary (rest of cases):*
 - $v > 200$ km/h and non isostatic bridge (always)
 - $v > 200$ km/h and isostatic bridge (where $L < 40$ m, or $f_{\text{torsion}} \leq 1.2f_0$, or $(v/f_0)_{\text{lim}}$ do not fulfil the requirements of tables F1/F2.

As a result of the dynamic analysis, the following value is obtained:

$$\varphi'_{\text{din}} = \max \left| \frac{y_{\text{din}}}{y_{\text{est}}} \right| - 1. \quad (22)$$

This factor will be calculated for all real trains (RT) or for HSLM trains. The maximum dynamic effect will be finally calculated as:

$$(1 + \varphi'_{\text{din}} + \varphi''/2) \times (\text{HSLM } \acute{o} \text{ RT}), \quad \acute{o} \quad \Phi \times (\text{LM71} + \text{SW/0})$$

Finally, it should be mentioned that for the case of bridges with two tracks, the Eurocode prescribes the dynamic analysis only in one of them.

6 EVALUATION OF THE DYNAMIC VEHICLE–STRUCTURE INTERACTION IN SIMPLY SUPPORTED BRIDGES

6.1 Scope of the study

The object of this application is to evaluate the effective reduction which is obtained, with respect to the dynamic analysis made without considering the vehicle–structure interaction, such

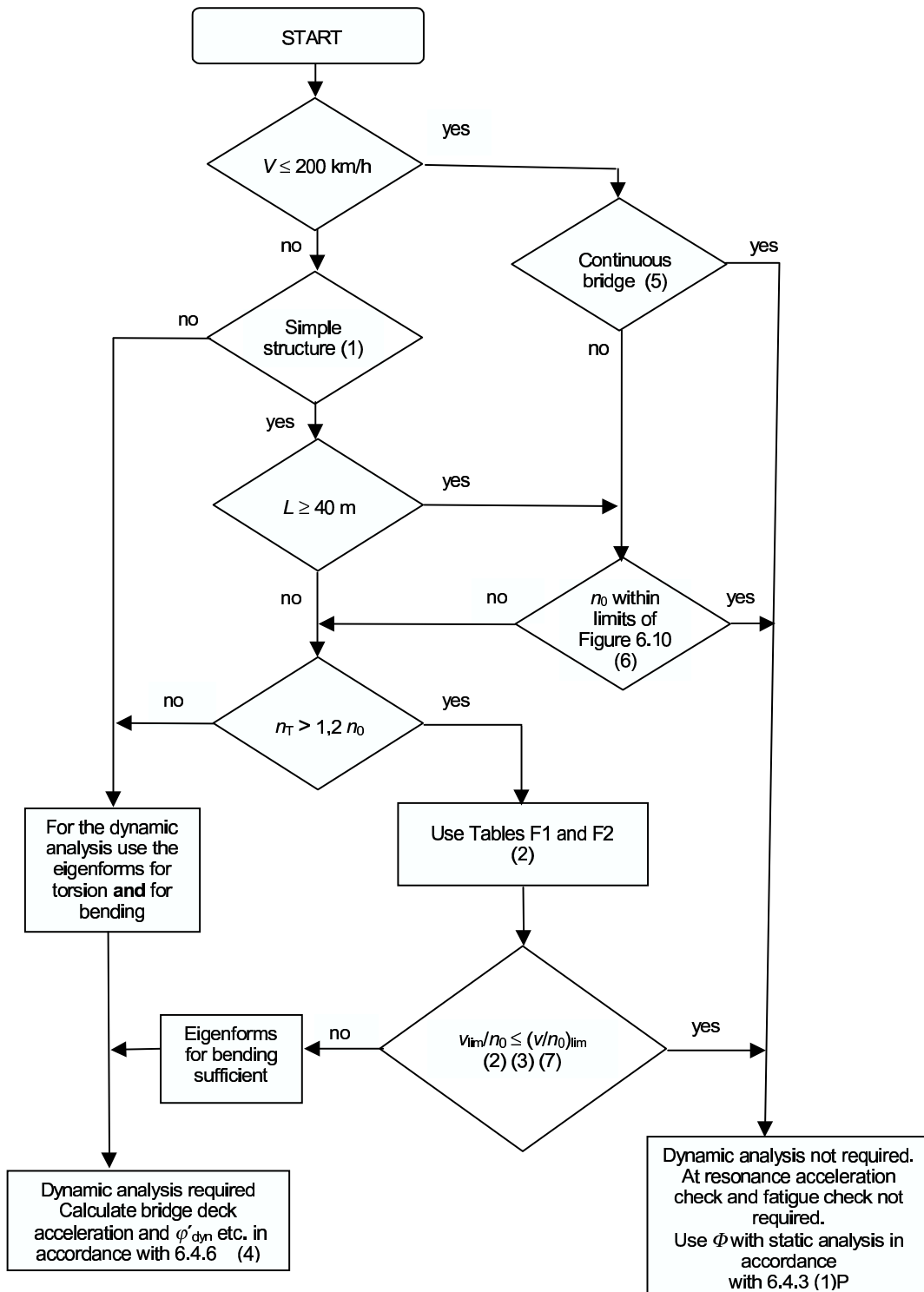


Figure 23. Flow chart of the new Eurocode 1 [22] for determining whether a dynamic analysis is required

as the models based on series of harmonics or models of single moving loads, more common in engineering practise.

The calculations are based on a modal analysis considering only first mode of vibration, without shear deformation. A model of moving loads is compared to the model with interaction proposed in the section 14. Time integration has been carried out using the trapezoidal rule. Isostatic bridges of spans (l) between 10 and 40 m have been considered, with characteristics according to the catalogue of isostatic bridges from [7]. The velocity sweep is (120 – 420) km/h, with $\delta v = 2.5$ km/h. The trains employed are the Ice2, Eurostar and Talgo AV, defined in [13], with damping rates $\zeta = 0.5\%$, $\zeta = 1\%$, $\zeta = 1.5\%$ and $\zeta = 2.0\%$. The calculations have been performed with the computer program [5].

The analysis results, as was predictable, show a significant reduction of the maximum displacements and accelerations for models with interaction. Some of the results obtained are included in table 2.

6.2 Results

In view of the results shown, one may conclude in first place that the moving load models clearly overestimate, in general terms, the response in accelerations and displacements of an isostatic structure; in comparative terms, the interaction models can reduce the maximum acceleration values in isostatic bridges up to 45% respect to acceleration obtained with single loads models.

Additionally, the dynamic response reduction, for the same hypothesis of span and damping, is greater for accelerations than for displacements, and the reduction increases as the line design speed is increased. Finally, it is also observed that the reduction of the response decreases when we increase the damping rate or the bridge span.

7 DYNAMIC RESPONSE OF PORTAL FRAMES FOR UNDERPASSES

7.1 Model used for calculations

For isostatic beams with load trains, the first vibration mode is the one that has a preponderant importance. For a beam with a span with an *general* support conditions, the fundamental eigenfrequency of the first vibration mode can be expressed as:

$$\omega = \left(\frac{\pi/\beta_1}{l} \right)^2 \sqrt{\frac{EI}{\bar{m}}} \quad (\text{rad/s}) \quad (23)$$

with $\beta_1 = 1$ for isostatic beams and $\beta_1 = 0.6642$ for fully restrained beams. The deck of a portal frame underpass is somewhere in between both cases, as it can be assumed to a certain extent to behave like a beam with flexible restraints in its ends. This way, the parameter obtained from the equation (7) for the first vibration mode of the portal frame will tend to π or to $\pi/0.6642$, for the more flexible or stiffer piers, respectively. The relevant parameters of the deck are, in a first approach, its length l , bending stiffness EI and linear mass \bar{m} . Considering a certain length of an *equivalent isostatic beam*, and fitting the rest of parameters, this beam could provide identical fundamental eigenfrequency as the frame, and therefore would have similar dynamic response.

220		$\zeta=0.5\%$		$\zeta=1\%$		$\zeta=2\%$		$\zeta=4\%$	
L [m]		desp	acel	desp	acel	desp	acel	desp	acel
5		-25%	-35%	-15%	-25%	-10%	-20%	-5%	-10%
10		-30%	-35%	-20%	-25%	-10%	-15%	-5%	-10%
15		-25%	-45%	-15%	-35%	-5%	-20%	0%	-15%
20		-10%	-20%	-5%	-15%	0%	-10%	0%	-5%
25		-10%	-35%	-5%	-25%	0%	-10%	0%	-5%
30		0%	-15%	0%	-5%	0%	0%	0%	0%
40		0%	-10%	0%	-5%	0%	-5%	0%	0%

250		$\zeta=0.5\%$		$\zeta=1\%$		$\zeta=2\%$		$\zeta=4\%$	
L [m]		desp	acel	desp	acel	desp	acel	desp	acel
5		-25%	-35%	-15%	-25%	-10%	-20%	-5%	-10%
10		-30%	-35%	-25%	-25%	-15%	-15%	-5%	-10%
15		-25%	-45%	-15%	-35%	-5%	-20%	-5%	-15%
20		-10%	-20%	-5%	-15%	0%	-10%	0%	-5%
25		-10%	-35%	-5%	-25%	0%	-10%	0%	-5%
30		-5%	-15%	0%	-15%	0%	-5%	0%	-5%
40		0%	-10%	0%	-5%	0%	-5%	0%	0%

270		$\zeta=0.5\%$		$\zeta=1\%$		$\zeta=2\%$		$\zeta=4\%$	
L [m]		desp	acel	desp	acel	desp	acel	desp	acel
5		-25%	-35%	-15%	-25%	-10%	-20%	-5%	-10%
10		-30%	-35%	-25%	-25%	-15%	-15%	-5%	-10%
15		-25%	-45%	-20%	-35%	-10%	-20%	-5%	-15%
20		-10%	-20%	-5%	-15%	0%	-10%	0%	-5%
25		-10%	-35%	-5%	-25%	0%	-10%	0%	-5%
30		-5%	-15%	0%	-15%	0%	-5%	0%	-5%
40		0%	-10%	0%	-5%	0%	-5%	0%	0%

300		$\zeta=0.5\%$		$\zeta=1\%$		$\zeta=2\%$		$\zeta=4\%$	
L [m]		desp	acel	desp	acel	desp	acel	desp	acel
5		-25%	-35%	-15%	-25%	-10%	-20%	-5%	-10%
10		-30%	-35%	-25%	-25%	-15%	-15%	-5%	-10%
15		-30%	-45%	-20%	-35%	-10%	-20%	-5%	-15%
20		-10%	-20%	-5%	-15%	0%	-10%	0%	-5%
25		-10%	-35%	-5%	-25%	0%	-10%	0%	-5%
30		-5%	-15%	0%	-15%	0%	-5%	0%	-5%
40		0%	-10%	0%	-5%	0%	-5%	0%	0%

350		$\zeta=0.5\%$		$\zeta=1\%$		$\zeta=2\%$		$\zeta=4\%$	
L [m]		desp	acel	desp	acel	desp	acel	desp	acel
5		-25%	-35%	-15%	-25%	-10%	-20%	-5%	-10%
10		-30%	-35%	-25%	-25%	-15%	-15%	-5%	-10%
15		-30%	-45%	-20%	-35%	-10%	-20%	-5%	-15%
20		-15%	-20%	-10%	-20%	-5%	-15%	-5%	-10%
25		-20%	-35%	-15%	-25%	-5%	-15%	-5%	-10%
30		-10%	-15%	-5%	-15%	-5%	-10%	0%	-5%
40		-5%	-10%	0%	-10%	0%	-5%	0%	0%

375		$\zeta=0.5\%$		$\zeta=1\%$		$\zeta=2\%$		$\zeta=4\%$	
L [m]		desp	acel	desp	acel	desp	acel	desp	acel
5		-25%	-35%	-15%	-25%	-10%	-20%	-5%	-10%
10		-30%	-35%	-25%	-25%	-15%	-15%	-5%	-10%
15		-30%	-45%	-20%	-35%	-10%	-20%	-5%	-15%
20		-20%	-20%	-15%	-20%	-10%	-15%	-5%	-10%
25		-20%	-35%	-15%	-25%	-5%	-15%	-5%	-10%
30		-10%	-15%	-5%	-15%	-5%	-10%	0%	-5%
40		-5%	-10%	0%	-10%	0%	-5%	0%	0%

Table 2. Reduction obtained of maximum acceleration and displacement with interaction model with respect to the moving load model. $V_{max}^{line} = V_0 = 220, 250, 270, 300, 350$ and 375 km/h

With this philosophy, the analogy could be made in several different ways, either conserving the deck length l , \bar{m} , EI or its total mass $M = \bar{m}l$. This last option would ensure preserving the kinetic energy of vibration in the equivalent beam.

The differences between the dynamic behaviour of the frame and its equivalent beam could be improved fitting the assigned \bar{m} and EI proportionally, so that the ratio $\hat{k} = EI/\bar{m}$ between them remains unchanged. According to [4], this proportional variation would allow to maintain the first vibration frequency invariable, without modifying the critical speeds for resonance, that is, without modifying the form of the envelopes, decreasing or increasing the maximum displacements and accelerations. Therefore, the objective will be to first obtain a beam whose envelopes are as similar as possible to those of the frame, and then fitting (if necessary) \bar{m} and EI conserving their ratio (factor \hat{k}), to fit the maximum displacements and accelerations.

7.2 Verification of simplified model

In order to establish the most adequate model for an equivalent beam, four beams have been defined for each frame of given span l . The length of these equivalent beams (l_{eq}) is respectively l , $0.95l$, $0.90l$, and $0.85l$. The mass of the equivalent beams is calculated so as to conserve the total mass of the deck, $\bar{m}_{viga} = \bar{m}_{dintel}l/l_{eq}$. To obtain the equivalent beam bending stiffness EI , its first mode vibration frequency is adjusted to the frequency of the portal frame:

$$(EI)_{eq} = \frac{\omega_{marco}^2 \bar{m}_{viga} l_{eq}^4}{\pi^4} \quad (24)$$

The same damping rate has been considered within the frame and the beam, and neither shear deformation nor the contribution of the earth cover have been considered. This last aspect provides a more conservative evaluation, as the additional masses would decrease the maximum accelerations and displacements. Only the first vibration mode has been taken into account, and neither the effects of vehicle–structure interaction nor track irregularities have been considered. The structures selected for the analysis have been 4 representative underpasses of high speed railway line between Córdoba and Málaga, with spans of 8.5, 8.7, 9.8 and 15 m. Thus, a total of 20 beam or frame structures have been studied. The frames have been analysed with the finite element program [10], and the equivalent beams with the program [5]. For each structure, the envelopes of accelerations, displacements and impact factors Φ have been calculated with the 7 real trains specified in [13]. The calculations are performed for a range of velocities of (120 – 420) km/h, with $\delta v = 5$ km/h. This makes a total of 8540 dynamic analyses. Some of the results are included in figures 24, 25 and 26, showing the envelope of maxima for each velocity for the frame of length 8, 5m, with a representative train of each one of the three types of real trains mentioned in figure 17.

7.3 Discussion of results

The criterion for selection of the most appropriate equivalent beam model was the greatest similarity between the accelerations envelope with the frame. since this is the critical aspect in these structures. With this criterion, the equivalent beam of length l was selected. Hence

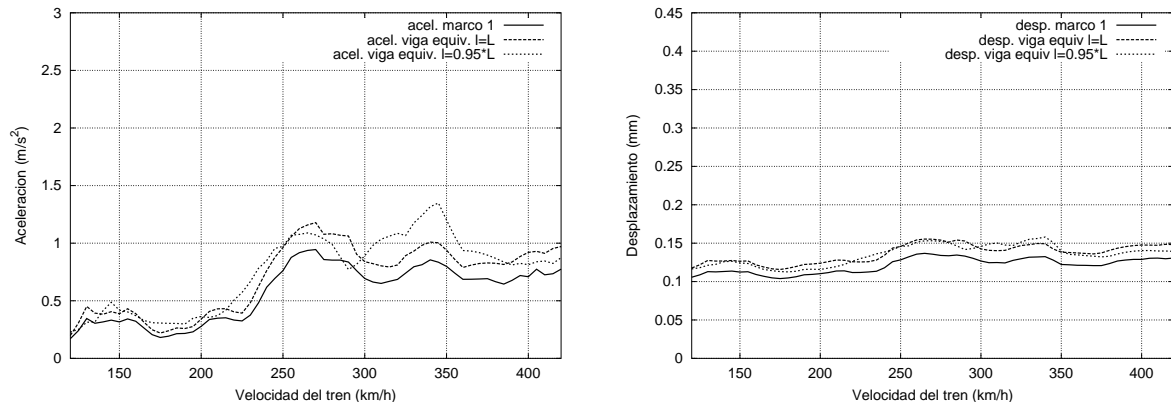


Figure 24. Envelopes of a_{max} , d_{max} from portal frame 1 and their equivalent beams of lengths l and $0.95l$ for Ave train

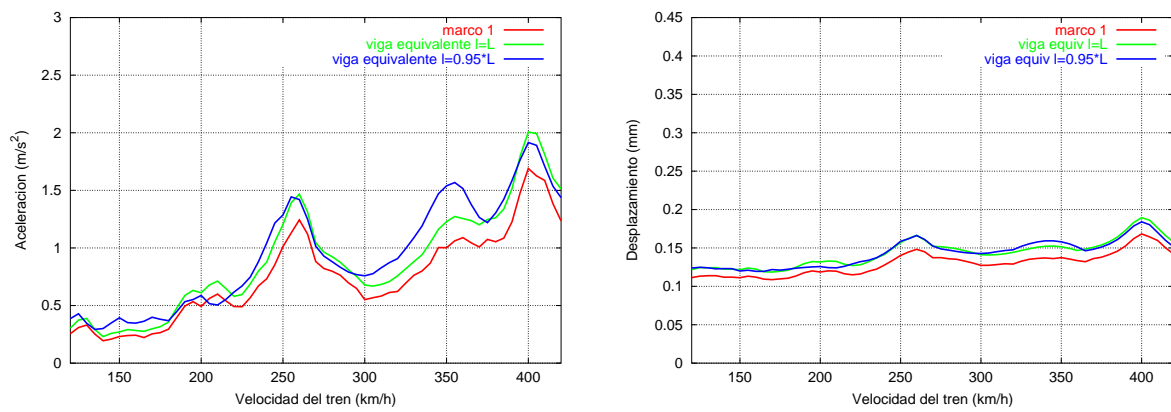


Figure 25. Envelopes of a_{max} , d_{max} from portal frame 1 and their equivalent beams of lengths l and $0.95l$ of train talgo AV

the equivalent isostatic beam will have the same span l , linear mass \bar{m} and damping rate ζ as the frame deck. The bending stiffness EI is obtained according to equation (24). In summary, other conclusions of this study are:

- It is possible to define an equivalent isostatic beam for the dynamic analysis of the usual frames in railroad underpasses, which conserve the form of envelopes of accelerations (a), displacements (δ) and impact factors (Φ).
- The isostatic equivalent beam yields, in practical terms, a conservative evaluation in critical design points, which are the circulation speeds where the maximum of the envelopes appears, for a , δ and Φ . In addition, the adjustment mentioned in the section 7.1 of the quotient $\hat{k} = \frac{EI}{\bar{m}}$ is not necessary.
- For non critical speeds, i.e. speeds at which the value of the response is not the maximum value, the results obtained for the equivalent beam are almost always greater than the results for the portal frame (figures 24, 25 and 26). However, it cannot be stated with absolute generality that the equivalent isostatic beam presents greater effects than those obtained for the frame for any non critical speed. Nevertheless, this aspect lacks relevance for the design of the structure.
- The frames which were studied are acceptable from a design point of view, their dynamic behaviour yields values within the acceptable limits: always $\Phi \leq 1$, and $a_{max,cdv} = 1.69 \text{ m/s}^2 \leq 0.35 \text{ g}$, limit established in [13, 22].

8 SENSITIVITY TO TIME-STEP IN TIME INTEGRATION OF MODES

Before making a modal analysis, it is convenient to estimate the most adequate time-step for the time integration. In order to illustrate the importance of this point, the maximum accelerations produced by a train Talgo AV in a isostatic bridge of 10 m span are studied (mechanical characteristics taken from the catalogue of bridges of [7]). Figure 27 compares the results analysed with a moving load model and modal analysis for different time-steps. The structural damping rate is $\zeta = 0.5\%$. The integration algorithm used is the trapezoidal rule, with fixed steps of $\delta t = 0.0001 \text{ s}$, $\delta t = 0.001 \text{ s}$, $\delta t = 0.01 \text{ s}$, $\delta t = 0.1 \text{ s}$ and $\delta t = 1 \text{ s}$.

It is possible to observe that, as the integration step δt is reduced, the results obtained converge to the solution which could be considered *exact*. In fact, the curves corresponding to steps $\delta t = 0.0001 \text{ s}$ and $\delta t = 0.001 \text{ s}$ are indistinguishable. However, for large values of δt (in this case: $\delta t = 1 \text{ s}$ and $\delta t = 0.1 \text{ s}$) the curve of maximum accelerations differs radically from this exact solution. This fact shows that with large time integration steps taking into account the structural vibration frequency ($f_0 = 8 \text{ Hertz}$), the dynamic behaviour of the bridge cannot be characterised adequately.

In [7] and [21] the criterion for performing a dynamic analysis is to consider only the modes with frequencies $f < 20 \text{ Hertz}$. In the last draft of the Eurocode-1 [22], this frequency limit has been increased to $f < 30 \text{ Hertz}$.

Following are summarised the results obtained in a study of the different recommendations that appear in the codes and in technical literature for the integration step in dynamic analysis

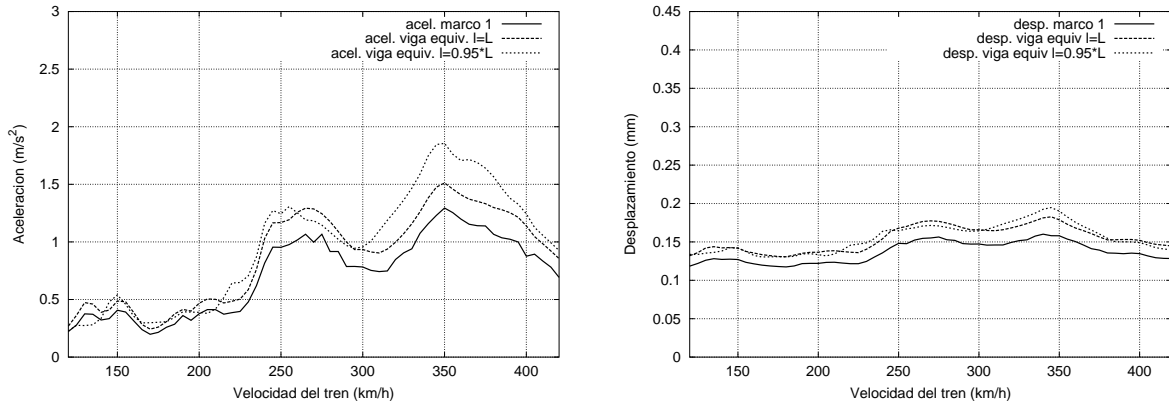


Figure 26. Envelopes of a_{max} , d_{max} from portal frame 1 and their equivalent beams of lengths l and $0.95l$ of train Ice2

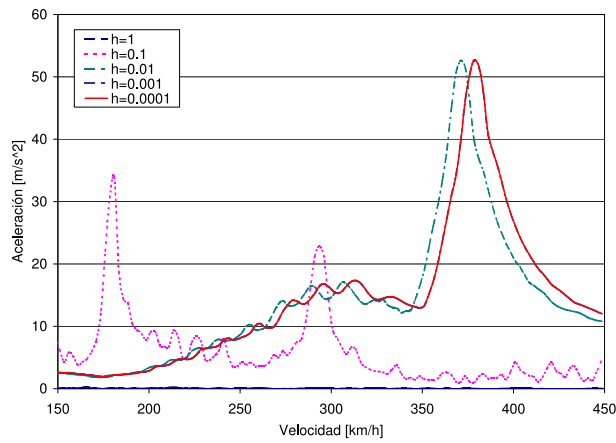


Figure 27. Maximum acceleration at centre of span, function of the speed of train Talgo AV, for different integration time-steps h . Simply supported beam with 10 m span from the catalogue of [7]. $\zeta = 0.5\%$.

of railroad bridges. According to [7], [6] and [25], the following integration steps h could be used:

- Determination of the integration step h based on the higher vibration frequency of the considered structure:

$$h_1 = \frac{1}{8f_{max}} \quad (25)$$

- Determination of the integration step h based on the minimum number of time intervals (in this case, two hundred) existing during the transit of an axis by the shortest span of the structure:

$$h_2 = \frac{L_{min}}{200 v} \quad (26)$$

- Determination of the integration step h based on the number n of vibration modes considered and the length of the shortest span of the structure:

$$h_3 = \frac{L_{min}}{4 n v} \quad (27)$$

- Time-step h independent from other parameters:

$$h_4 = 0.001 \text{ s} \quad (28)$$

- Fixed integration step h that acts as filter of frequencies higher than 50 Hertz (models of direct time integration of the structure):

$$h_5 = 0.002 \text{ s} \quad (29)$$

Figure 28 shows, for a reference case, the maximum acceleration results obtained² according to the different alternatives for selection of time-step outlined above.

The results obtained for the steps h_2 , h_4 and h_5 are similar. The determination of the optimal one between these three steps will depend on other factors like, for example, the convenience of using the same integration step in a speed sweep—in this case we recommend the use of h_2 with $v = v_{max}$ —or the optimisation of the total time for integration.

9 SIMPLIFIED EVALUATION OF TORSION IN REAL VIADUCTS

A simplified method to evaluate the torsion effects in railroad bridges is proposed in [8]; As a conservative design envelope, this committee considers the linear superposition of the associated dynamic effects of bending and torsion, studying separately both effects.

This way, two independent analyses would be performed (bending and torsion), determining the maximum responses in absolute value (R_f and R_t , respectively) for the variables under study, usually displacements or accelerations. The simplest method of combination is proposed, obtaining the total response R_{total} as the direct absolute sum of the individual responses: $R_{total} = R_f + R_t$. It should also be possible to consider—although this possibility

²The results presented in this section have been obtained with a moving load model, with integration of the resulting equations using the trapezoidal rule.

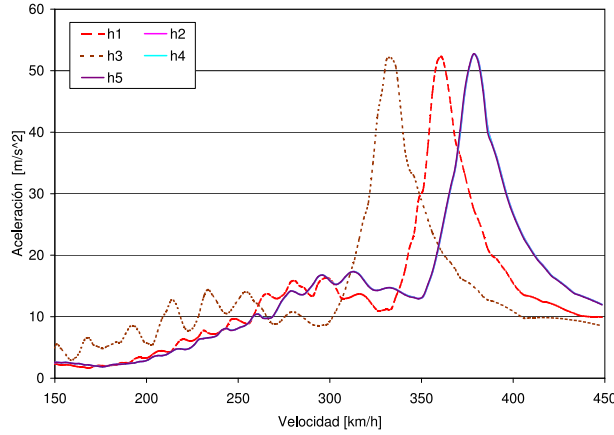


Figure 28. Maximum acceleration at centre of span, function of the train speed of a Talgo AV, for different integration time-steps h . Simply supported beam from the catalogue of bridges from ERRI D214 with 10 m span. $\zeta = 0.5\%$.

is not mentioned in [8]— the application of the square root of the sum of squares (SRRS) for the combination of these two actions, whenever the frequencies of both vibrations are sufficiently separate ($\omega_i - \omega_j \geq 20\% \omega_i$). We remark that the use of this hypothesis allows the use of simplified methods of analysis.

In this section we report an evaluation of the adjustment of this simplified method, applying it to two bridges of the future high speed line between the cities of Córdoba and Málaga. For the analysis, six high speed European trains have been considered (Virgin, Ave, Ice2, Etr-y, Eurostar and Talgo AV). The structural sections corresponding to these bridges are representative of those being built in modern railroad bridges for HS lines: hollow slab and box section. The bridges were considered as simply supported in bending, but with a full restraint for torsion in the supports.

Bridge	Box section	Hollow slab section
span [m]	46	23.5
\bar{m}_c [*] [kg/m ³]	3804	3840
Area [m ²]	10.50	10.22
I_x [m ⁴]	21.03	3.35
I_y [m ⁴]	120.06	99.66
I_ω [m ⁴]	27.03	7.64
E [MPa]	36149.6	36149.6
G [MPa]	15062.3	15062.3
ζ	2%	2%

[*] The material density \bar{m}_c has been modified to include the effect of non-structural dead load.

Table 3. Characteristics of bridges analysed for bending and torsion.

The computer program employed for these dynamic analyses is [10]. In order to compare the proposal of simplified evaluation of [8] with the application of a complete three-dimensional model, the following process has been followed:

- Dynamic analysis to evaluate the effects due to longitudinal bending.
- Dynamic analysis to evaluate the effects due to transversal torsion (only torsion). These results have been obtained by two different ways:
 1. From a bending analysis, applying the existing proportionality between the maximum accelerations curves associated to torsion and bending.
 2. With the results obtained directly from the finite elements model used in torsion.
- Complete dynamic analysis combining bending and torsion.

The comparison criterion that has been established is to relate the results obtained with the simplified method (direct sum of the absolute maximum of response obtained from the *only bending* analysis plus the maximum obtained from the *only torsion* analysis) with the maximum corresponding to the complete model (bending and torsion). In addition, the possibility of combining the hypotheses of *only bending* and *only torsion* with method SRSS (square root of the sum of squares) has been evaluated.

The table 4 shows the maximum values of the resulting accelerations in the centre of the span for the all the different hypotheses. It also includes the existing deviation between the simplified model and the complete one considering simultaneous combination of bending and torsion.

Bridge	Box section	Hollow slab section
Max. acceleration only bending [m/s ²]	1.27	3.32
Max acceleration only torsion [m/s ²]	0.05	0.28
Max. acceleration combined model [m/s ²]	1.29	3.41
Max. acceleration simplified model [m/s ²]	1.32	3.61
Deviation simplify model vs. combined	≈ +2.2%	≈ +5.72%
Max acceleration max. SRSS [m/s ²]	1.27	3.33

Table 4. Summary of analysis results obtained for different alternatives for considering torsion, as only bending, only torsion and as a combined model.

From the study of the results obtained, the following conclusions can be extracted:

- For the sections studied, the simplified method proposed in [8] for the analysis of the phenomena of combination of bending and torsion is valid. The simplified method modified with the use of SRSS combination, is not on the safety side. However, this fact is due to the reduced value of torsion accelerations, and the deviation is small.

- For structural sections with large torsional stiffness GI_ω , the deviation of results obtained with the simplified method and those obtained with the combined model of bending–torsion is almost insignificant; thus, this simplified method is recommended.
- For sections with reduced torsional stiffness (for example, hollow slabs, open sections), the existing deviation of results between the simplified models and those of interaction bending–torsion is more significant (in the case studied, near 5%). In case of being near to the limits of accelerations or total displacements established by the code, it is recommended to obtain more precise results using combined models which include bending and torsion.

This study has been limited to bridges with closed boxed sections and hollow slabs. There are sections, of common use in other countries, where the torsion effects are more important. In [17] the analysis of a composite steel-concrete bridge with open boxed section and with cross lower bracing are detailed; in those bridges the torsion accelerations are more relevant as design limitations.

10 CONCLUSIONS

Following we summarise some final remarks:

- The design of high speed railroad bridges, because of the real possibility of resonance, require consideration of the dynamic vibration under moving loads. For this purpose several models of analysis are described in this article, of smaller or greater complexity.
- It is very important to apply these dynamic analysis methods in applied research to improve our knowledge about the most determinant factors for the dynamic response of the bridges from the project point of view, as well as to be able to develop engineering design methods and codes which are sufficiently practical, secure, and simple to use.
- The new draft of IAPF [13] and the final draft of Eurocode 1 of actions in bridges [22] cover adequately this necessity of dynamic analysis for the high speed lines.
- The dynamic analysis methods have been applied to some representative design problems, obtaining as a result the validation of simplified methodologies which may help in a more reliable evaluation of engineering designs.

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