Resonant effects in short span high speed railway bridges: modelling and design issues

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ABSTRACT: Trains at speeds above 200 km/h may induce resonance in bridges or viaducts. This fact implies the necessity of sufficiently detailed dynamic calculations, which has been included in the latest engineering codes of practise (FS 1997; prEN 1991-2 2002; IAPF 2001).
Various mathematical models are available for dynamic analysis of trains of moving loads, generally through direct time integration of the equations of motion. Some simplified analytical methods are also available (ERRI D214 (a) 1998) which provide directly dynamic envelopes. However, these methods are only valid for simply-supported beams. In some cases it may be necessary to employ complex models with vehicle-bridge interaction. In this paper we discuss the applicability of the above methods for calculation of railway bridges. Following, two practical studies applying the above techniques are presented. For portal frame underpasses, a simplified method of calculation is justified which may greatly simplify engineering calculations. Finally, we also present results of a study evaluating the relative reduction in dynamic effects which may be obtained by considering vehicle-bridge interaction.

1 INTRODUCTION

The construction of new high speed railway infrastructure constitutes currently in many European nations one of the major civil engineering efforts. Concerning railway bridges, one of the main design issues has always been the dynamic (moving) loads, for which basic solutions have been described by (Timoshenko and Young 1955), being discussed fully in (Fryba 1972; Fryba 1996).

Most engineering design codes for railway bridges have followed the approach of a dynamic factor proposed in (UIC 1979), which takes into account the dynamic effect of a single moving load and yields a maximum dynamic increment of $\varphi' = 132\%$ for a track without irregularities. (To be taken into account via a factor $\Phi = 1 + \varphi'$ which multiplies the static effects.) This approach does not cover the possibility of resonant response of the bridge due to a periodic array of moving loads, as this phenomenon does not appear for train speeds below 200 km/h.

However, resonance is all too real for high speed railways, and its effects may surpass largely that of a single moving load. An illustrative example is documented with experimental measurements and model predictions in (Domínguez Barbero 2001) for the Spanish AVE crossing the Tagus bridge. New European codes include the need for dynamic calculations covering resonant behaviour (FS 1997; prEN 1991-2 2002; IAPF 2001).

For the purpose of dynamic analysis of railway bridges several methods are available. The simplest ones are based on sums of harmonic terms, which provide bounds for dynamic response (ERRI D214 (a) 1998), with application limited to isostatic (simply-supported) bridges. Alternatively, direct dynamic calculations may be performed on full or reduced models with or without vehicle-bridge interaction. Some of these models are discussed in the following sections.

In the project of an important or singular bridge or viaduct (with a proportionally high budget) the application of the necessary dynamic calculation methods poses no important problems. However, it appears that also some very simple railway bridges with common structural types may require sophisticated methods for analysis. This may be the case, for instance, of portal frame structures typical of railway underpasses, which are statically redundant, and in principle would require a dynamic calculation involving several vibration modes. It is also the case of some short-span simply-supported bridges, in which the acceleration of the deck may be surprisingly high.

In this paper, direct dynamic integration methods are applied to such simple, short-span bridges in order to gain improved knowledge regarding their dynamic behaviour. In the case of simply supported beams, the effect of taking into account vehicle-structure interaction on the resonant peak response is evaluated for a representative series of European high-speed train models. For the case of portal frame structures, the possibility of analysing them with a simpler model of an equivalent simply-supported beam is explored and compared to a full multi-mode dynamic structural analysis.

2 MODELS BASED ON DIRECT INTEGRATION WITH MOVING LOADS

This class of methods is based on the direct time integration of the dynamic equations of the structure, subject to a given train of moving loads. The structural model may be studied either by the complete (discretized) system with N d.o.f., or by a prior modal analysis and reduction to $n \ll N$ significant eigenmodes. In turn, the modal analysis may be performed on a discretized approximate system (e.g. by finite elements) or alternatively, when analytical solutions are available, directly on the continuous system.

2.1 *Analytical mode extraction*

The simplest case is that of a slender (no shear deformation) simply-supported beam. The modes of vibration and associated eigenfrequencies are:

$$\phi_n(x) = \sin\left(n\pi\frac{x}{l}\right); \quad \omega_n = \left(n\frac{\pi}{l}\right)^2\sqrt{\frac{EI}{\overline{m}}}, \qquad (1)$$

where l is the span, EI the bending stiffness, and $\overline{m}$ the mass per unit length.

Analytical mode extraction is not possible in general for statically redundant structures, except for some special cases, such as some continuous beams (Fryba 1972). A similar such case is that of a portal frame consisting of a horizontal deck fully tied to two vertical members. The eigenfrequencies are given by

$$\omega_1 = \left(\frac{b}{l_d}\right)^2\sqrt{\frac{E_dI_d}{\overline{m}_d}}, \qquad (2)$$

where l_d is the span of the deck, E_dI_d its bending stiffness, $\overline{m}_d$ its mass per unit length, and b is obtained by solving the following nonlinear equation:

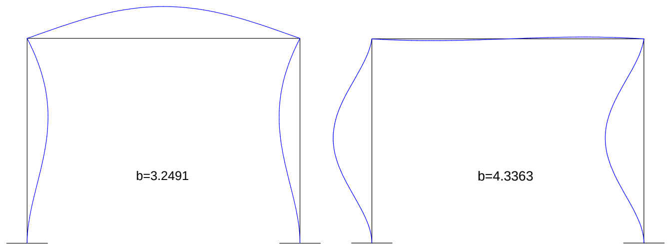
$$\frac{k_p(1-\cosh(k_ib)\cos(k_ib))}{\cosh(k_ib)\sin(k_ib)-\sinh(k_ib)\cos(k_ib)} + \frac{1-\cosh b\cos b}{(\cosh b+1)\sin b-(\cos b+1)\sinh b} = 0,$$

with

$$k_p = \sqrt[4]{\frac{I_d^3}{I_h^3}\frac{\overline{m}_d}{\overline{m}_h}}; \quad k_i = \frac{l_h}{l_d}\sqrt[4]{\frac{I_d}{I_h}\frac{\overline{m}_h}{\overline{m}_d}}.$$

In this last equation subindex h refers to the vertical members of the portal frame.

In figure 1 the first two eigenmodes are shown for a real case of a portal frame in a high speed line (Goicolea et al. 2001), which will be further discussed in section 5.



(a) 1st. mode (b) 2nd. mode

Figure 1: First two eigenmodes of a portal frame for an underpass of a high speed railway line. (parameter b corresponds to equation (2))

Once the eigenmodes $\phi_i(x)$ are determined, the dynamic equation for each mode amplitude y_i under a train of loads F_k moving at speed v (Fig. 2) is written as

$$M_i\ddot{y}_i + 2\zeta_i\omega_iM_i\dot{y}_i + \omega_i^2M_iy_i = \sum_{k=1}^{n_{\text{axles}}} F_k\left\langle\phi_i(vt-d_k)\right\rangle, \qquad (3)$$

where M_i is the modal mass, ζ_i the modal damping ratio, and the notation $\langle\phi(\bullet)\rangle$ has the following meaning:

$$\langle\phi(x)\rangle = \begin{cases}\phi(x) & \text{if } 0 < x < L\\ 0 & \text{otherwise.}\end{cases} \qquad (4)$$

2.2 *Finite element models*

Finite element techniques perform a (semi-) discretisation in spatial coordinates which is applicable to any type of structure, including non linear type of behaviour. As a result, a discrete N–d.o.f. system of equations is obtained:

$$\mathbf{M}\ddot{\mathbf{d}} + \mathbf{C}\dot{\mathbf{d}} + \mathbf{K}\mathbf{d} = \mathbf{f}(t), \qquad (5)$$

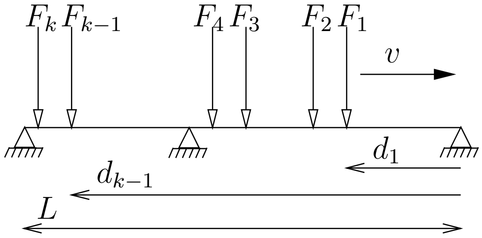


Figure 2: Response for a load train F_k

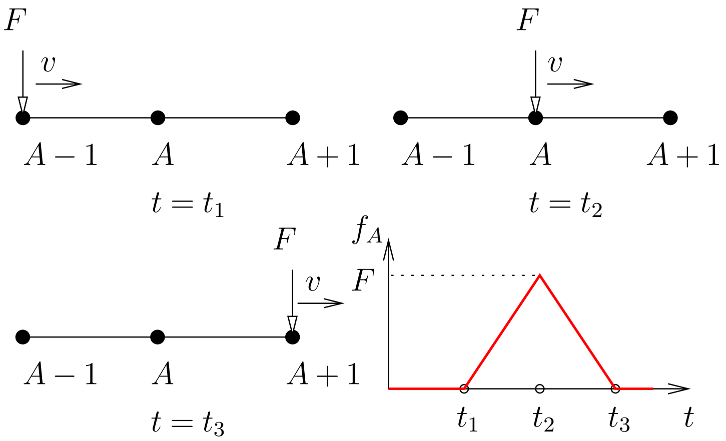


Figure 3: Definition of load in node A for moving load F in finite element model

where $\mathbf{M}$, $\mathbf{C}$, $\mathbf{K}$ are the mass, damping and stiffness matrices respectively, $\mathbf{f}(t)$ the load vector (from moving loads), and $\mathbf{d}$ the vector of nodal displacements.

In order to integrate these equations in time, generally a modal analysis and reduction leading to a reduced number of significant eigenmodes $n \ll N$ is performed, arriving to uncoupled equations which are integrated by standard time integration techniques such as the β-Newmark method.

The simplest procedure to represent the load train is to apply load pulse time histories for each node, depending on the time of arrival and the discretisation (Fig. 3).

3 MODELS BASED ON HARMONIC SERIES

These models take advantage of the fact that for simply supported beams the modes of vibration are harmonic functions (1). Furthermore, generally only the first mode of vibration need be considered. Following equation (3), the response to a series of moving loads will be a sum of (damped) harmonic terms.

Based on different simplifications, it is possible to establish analytical expressions which provide bounds on the dynamic response of the beam. A common feature in these models is that these analytical expressions make use of the *dynamic signature* of the train. This is a function of special relevance in the intuitive interpretation of the response of a given train, which defines its aggressiveness in relation to the dynamic effects on a bridge. Two such models have been proposed in (ERRI D214 (a) 1998): the DER model (based on the decomposition of the excitation in resonance) and the LIR model (based on the residual vibrations after each load leaves the bridge). A slight modification of LIR called IDP has been proposed in (Domínguez Barbero 2001).

As an example we cite the expression of the LIR method for maximum acceleration at mid-span, Γ, obtained as a product of the following factors:

$$\Gamma = C_{\text{acc}} \cdot A(r) \cdot G(\lambda), \qquad (6)$$

where $C_{\text{acc}} = 1/M$ is a constant (equal to the inverse of the total mass of the span), $\lambda = v/f_0$ (*wavelength*),

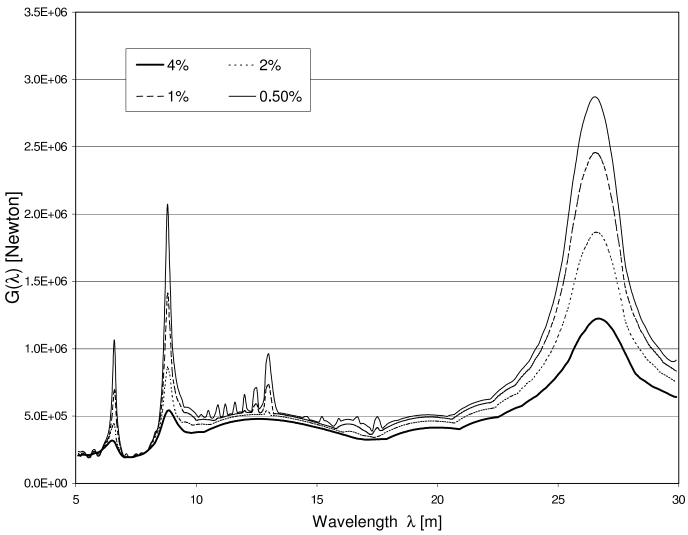


Figure 4: Dynamic signature of ICE2 train using LIR method

with v the train speed and f_0 the eigenfrequency (Hz) of the first mode, and $r = \lambda/(2l)$, l being the span. The remaining terms are defined by:

$$A(r) = \frac{r}{1-r^2}\sqrt{\mathrm{e}^{-2\zeta\frac{\pi}{r}} + 1 + 2\cos\left(\frac{\pi}{r}\right)\mathrm{e}^{-\zeta\frac{\pi}{r}}} \qquad (7)$$

$$G(\lambda) = \max_{i=1}^{N}\left\{\left[\sum_{x_1}^{x_i} F_i \cos(2\pi\delta_i)\,\mathrm{e}^{-2\pi\zeta\delta_i}\right]^2 + \left[\sum_{x_1}^{x_i} F_i \sin(2\pi\delta_i)\,\mathrm{e}^{-2\pi\zeta\delta_i}\right]^2\right\}^{1/2}. \qquad (8)$$

In these expressions ζ is the damping ratio, x_i are the distances of each of the N axles with corresponding load F_i to the first axle of the train, and $\delta_i = (x_i - x_1)/\lambda$.

The term $G(\lambda)$ in equation (8) is the so-called *dynamic signature*. It depends solely on the axle load distribution of the train and the damping ratio to be considered. Each train has a characteristic dynamic signature, which is obtained independently of the mechanical characteristics of the bridge. As an example, the dynamic signature of ICE2 high speed train is shown in figure 4 for various damping values.

The term $A(r)$ in equation (7) is the so-called dynamic influence line of the bridge, which defines the bridge response as a function of r (itself in turn linearly related to speed v), depending solely on the characteristics of the bridge: span (l), first eigenfrequency (f_0), and damping (ζ).

A similar expression to (6) is applicable to the displacement response of the deck, just by changing the constant to $C_{\text{disp}} = C_{\text{acc}}/(2\pi f_0)^2$.

The use of these methods allows for a very simple analytical evaluation, for a given bridge, of the critical resonance speeds for which the deck response is a maximum, without being necessary to perform a dynamic calculation with time integration. However, the drawback is their limited applicability to isostatic bridges, thus ruling out many practical cases.

4 MODELS WITH VEHICLE-BRIDGE INTERACTION

For a more general case, these models may include the stiffness and damping represent the primary and secondary stiffness and damping elements, as well as the mass and (rotatory) inertia of bogies and vehicle box. The effect of these is not only to introduce additional d.o.f.'s, but also to couple the motion of points under successive axles. In many cases the main effects of vehicle interaction with railway bridges may be adequately captured with simplified interaction models as proposed in (ERRI D214 (e) 1999), which do not consider rotation of the vehicle boxes.

A schematic representation of the model employed in this work and of the variables employed is shown in figure 5. The following equations are obtained for each mode of vibration $(i = 1 \ldots n)$:

$$M_i \, \ddot{y}_i + C_i \, \dot{y}_i + K_i \, y_i = \sum_{j=1}^{k} \langle \phi_i(d^j_{rel}) \rangle \left(g \, m^j + m^j_a \, \ddot{q}^j \right) . \tag{9}$$

Also, for each interaction element $(j = 1 \ldots k)$:

$$m^j_a \ddot{q}^j + k^j \left[q^j - \sum_{i=1}^{n} y_i \, \langle \phi_i(d^j_{rel}) \rangle \right]$$
$$+ c^j \left[\dot{q}^j - \sum_{i=1}^{n} \dot{y}_i \langle \phi_i(d^j_{rel}) \rangle - \sum_{i=1}^{n} y_i v \langle \phi'_i(d^j_{rel}) \rangle \right] = 0 \tag{10}$$

In equations (9) and (10) the notation $\langle \phi(\bullet) \rangle$ corresponds to that defined previously in (4). Additionally, the term d^j_{rel} refers to the relative position of element j on the bridge; taking the origin of time $t = 0$ when the head of the train enters the bridge ($x = 0$), $d^j_{rel} = vt - d^j$.

Finally, the equations may be integrated in time with standard time integration techniques, such as β-Newmark. A full description of this model is contained in (Domínguez Barbero 2001).

5 DYNAMIC RESPONSE OF PORTAL FRAMES

A portal frame is a statically redundant structure. In principle, for its dynamic analysis the direct time integration methods described in sections 2 or 4 should be employed, including several modes of vibration for the structure. The simpler models of section 3 are precluded since they only apply to simply-supported beams. When portal frames are embedded in an embankment they may also have earth on the sides or even on top of the deck. The proper analysis of earth-covered frames is fairly complex, however often the earth may be considered more simply as a non-structural added mass, which contributes to reduce the dynamic effects.

From an engineering point of view these requirements are rather inconvenient: portal frames are very simple structures, employed routinely for railway underpasses, with a correspondingly low budget for calculations.

The object of the work reported here has been to validate a simple model which would be suitable for the dynamic evaluation of such portal frames, without necessitating multi-mode direct time integration. The driving idea may be motivated by inspection of figure 1(a): the first mode shape of the deck is not too dissimilar to that of a simply supported beam. Also, figure 1(b) suggests that the second mode of the frame will not contribute much to deck vibrations. Hence, it could be possible to find an *equivalent beam* with fictitious mass, length and stiffness. This equivalent beam should possess similar dynamic characteristics as the frame deck, and hence exhibit a *dynamic envelope* of similar shape and equal or greater to that of the real frame. (This dynamic envelope is defined as the maximum dynamic effect for each train speed v, e.g. for accelerations the function $a_{\text{max}}(v)$.) For this purpose the equivalent beam cannot be just (trivially) assigned the mechanical properties of the deck, as it is clear that the edge restraints transmit some vibration energy to the vertical elements of the frame (see Fig. 1(a)), which must be taken into account.

A first observation is that for a simple beam with partial rotation restraint at the supports the first eigenfrequency may be expressed as

$$\omega_1 = \left(\frac{\pi}{\beta_1 l} \right)^2 \sqrt{\frac{EI}{\overline{m}}} \quad \text{(rad/s)}.$$

This expression, compared to (1) for $n = 1$, defines an equivalent length of a simply-supported beam, $l_{\text{eq}} = \beta_1 l$. For a simply-supported beam $\beta_1 = 1$, and $\beta_1 = 0.6642$ for fully-restrained supports. For a frame deck the partial restraint suggests an equivalent length between those two extremes.

The dynamic parameters of the equivalent beam to be obtained are the span l_{eq}, the mass per unit length $\overline{m}_{\text{eq}}$ and the bending stiffness $(EI)_{\text{eq}}$. For the equivalent lengths four different values have been evaluated: $l_{\text{eq}} = 0.85l,\ 0.90l,\ 0.95l,\ 1.0l$. For each case, the mass is assigned as $\overline{m}_{\text{eq}} = \overline{m}_{\text{deck}}/l_{\text{eq}}$. Finally, the bending stiffness is obtained such that the first eigenfrequency of the portal frame is maintained:

$$(EI)_{\text{eq}} = \frac{\omega^2_{\text{frame}} \, \overline{m}_{\text{eq}} \, l^4_{\text{eq}}}{\pi^4} \tag{11}$$

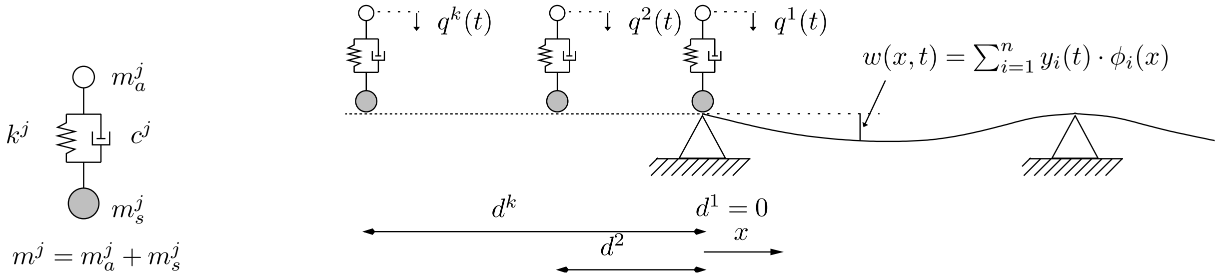


Figure 5: Vehicle-bridge interaction model employed

For the object of this work four portal frame underpasses of a high speed railway line to be constructed shortly were selected (PROINTEC 2001), with deck spans of 8.5, 8.7, 9.8 and 15 m. The calculations were carried out for the seven European high speed trains defined in (IAPF 2001): AVE, EUROSTAR 373/1, ETR-Y, ICE-2, TALGO-AV, THALYS, VIRGIN, for speeds between 120 and 420 km/h in steps of 5 km/h. This makes a total of 1708 dynamic calculations for frames, carried out with a modified version of FEAP (Taylor 2000), and 6832 calculations for simply supported beams (Goicolea et al. 2001).

The results obtained comprise displacements and accelerations at various points of the deck. As a representative example, figure 6 shows the comparison of dynamic envelopes obtained for frame no. 1, compared to equivalent beam calculations. It is seen that the beam with $l_{\text{eq}} = l$ yields an excellent simplified model, reproducing accurately the shape of the dynamic envelope, always on the conservative side above the response of the frame, but not overly conservative. Similar results were obtained for all the other frame types.

The implication of these results is that, at least for portal frames which can be covered by the types here tested, a simplified calculation method of the type described in section 3 may be applied for design purposes.

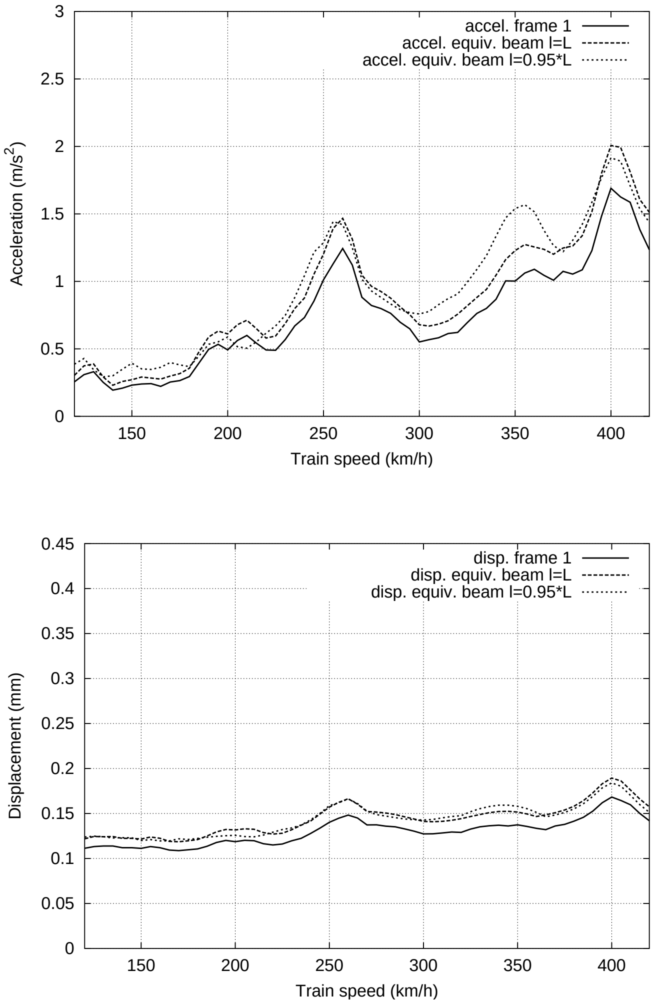


Figure 6: Comparison of acceleration and displacement results envelope between full frame model no. 1 and equivalent simply-supported beam models with $l_{\text{eq}} = 0.95\,l,\ 1.0\,l$

6 EFFECT OF VEHICLE-BRIDGE INTERACTION IN SIMPLY-SUPPORTED BRIDGES

In this section we summarise another application of interest in which the objective was to evaluate the reduction in dynamic effects corresponding to vehicle-bridge interaction, as compared to predictions of models with moving (fixed value) loads. Physically, this reduction is due to the energy of vibration transmitted to the vehicles, which is absent from the moving load models.

The motivation for this study is again a practical engineering design problem: the maximum deck accelerations obtained in short span beams (10 – 30 m) often exceed the maximum values permitted by the codes (i.e. $a_{\text{max}} \leq 3.5\text{m}/\text{s}^2$ for double track deck in (prEN 1991-2 2002)). Hence it is convenient to perform a precise enough calculation, without excessive conservatism. However, for such simple bridge types it is often out of the question (from the point of view of project engineers) to perform a complex dynamic analysis with vehicle-bridge interaction. It would be desirable to be able to take into account the interaction effect by a reduction coefficient or by a (fictitious) *added damping*.

The work reported, which is described in more detail in (Domínguez Barbero 2001), comprises simply-supported bridges with spans ranging between 5 and 40 m, following the catalogue of bridges defined for benchmark purposes in (ERRI D214 (a) 1998). Calculations have been performed using one mode of vibration, with the models described above (sect. 2.1, without interaction, and sect. 4, with interaction). Calculations were performed for HS trains ICE-2, EUROSTAR, and TALGO-AV, for which the data for vehicle dynamics were available to the authors, in each case for several damping ratios ranging between $\zeta = 0.5\%$ and 4%. The dynamic results envelopes were calculated for various ranges of speeds, starting at 120 and with maxima up to 420 km/h, in steps of 2.5 km/h.

An excerpt of the results is displayed in table 1, showing reductions in maximum dynamic effects of up to 45%. These reductions are more significant for the shorter spans, being also slightly larger for accelerations than for displacements. It may also be seen that for higher line speed, higher reductions are obtained.

270	ζ=0.5%		ζ=1%		ζ=2%		ζ=4%	
L [m]	disp	accel	disp	accel	disp	accel	disp	accel
5	-25%	-35%	-15%	-25%	-10%	-20%	-5%	-10%
10	-30%	-35%	-25%	-25%	-15%	-15%	-5%	-10%
15	-25%	-45%	-20%	-35%	-10%	-20%	-5%	-15%
20	-10%	-20%	-5%	-15%	0%	-10%	0%	-5%
25	-10%	-35%	-5%	-25%	0%	-10%	0%	-5%
30	-5%	-15%	0%	-15%	0%	-5%	0%	-5%
40	0%	-10%	0%	-5%	0%	-5%	0%	0%

(a) for speeds between 120 and 270 km/h

375	ζ=0.5%		ζ=1%		ζ=2%		ζ=4%	
L [m]	disp	accel	disp	accel	disp	accel	disp	accel
5	-25%	-35%	-15%	-25%	-10%	-20%	-5%	-10%
10	-30%	-35%	-25%	-25%	-15%	-15%	-5%	-10%
15	-30%	-45%	-20%	-35%	-10%	-20%	-5%	-15%
20	-20%	-20%	-15%	-20%	-10%	-15%	-5%	-10%
25	-20%	-35%	-15%	-25%	-5%	-15%	-5%	-10%
30	-10%	-15%	-5%	-15%	-5%	-10%	0%	-5%
40	-5%	-10%	0%	-10%	0%	-5%	0%	0%

(b) for speeds between 120 and 375 km/h

Table 1: Reduction of effects due to consideration of vehicle-structure interaction in simply-supported bridges

7 CONCLUSIONS

In summary we single out the following remarks.

- Due to the possibility of resonance, bridges in high speed railway lines must be designed taking into account the necessary dynamic analysis. This requirement has been introduced recently into the latest codes of engineering practise for railway bridges. A number of analysis methods exist with different degree of complexity, which have been discussed in this paper.

- Not enough knowledge is available at present regarding the dynamic and resonant behaviour of a number of structural types common in engineering practise. In some cases this may lead to the necessity of performing sophisticated dynamic analyses for the design of very simple structures.

- A simple design procedure is proposed for the calculation of dynamic effects in portal frames, based on the dynamic equivalence with a fictitious simply-supported beam. This procedure has been validated for a range of portal frames in a new HS railway line, and could be generalised to a wider range of cases.

- The reduction of dynamic effects in short-span simply supported bridges due to vehicle-bridge interaction has been evaluated. This proves to be a significant proportion (45%) of the values obtained without taking interaction into account.

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